Process of the Lockstitch Tightening and Optimisation of the Thread Working Conditions.
Part I. Dynamic Model of the Phenomenon

Abstract
During the formation of lockstitches in the sewing machine, the thread inserted into the stitch link by the needle undergoes extreme destruction as the result of the machine tool’s action. The stage of inserting the interlacement into the needle channel (known as stitch tightening), connected with the thread take-up working programme, is presumed to be the most destructive stage for the thread. In this paper, the model of the stitch tightening process in dynamic conditions is presented. It allows us to assess the location of the interlacement in the stitch link, as a function of the sewing process technological parameters, such as the machine rotary speed, the restraint forces of the needle thread and the bobbin hook thread, the geometrical characteristics of the stitch formation zone, the thread elastic features etc.

Key words: sewing machine, sewing thread, lockstitch, stitch tightening, interlacement.

Introduction
In the machine sewing processes, the destruction of the sewing thread in the stitch formation zone is one of the main technological disadvantages [2 - 6]. This problem has particular significance when the needle thread forms the lockstitch, because its destruction level resulting from the machine tool action is so high that it may lead to a reduction in both the sewing process efficiency (technological thread breakage [7]), and the utility & durability of the clothing article (seam strength).

It has been stated that the number of the thread loading cycles, as expressed by the dynamic loading quotient [4, 8] and the dynamic force magnitude generated in the thread within the single machine working cycle, determines the thread destruction level in the stitch formation zone. Both the above-mentioned factors are closely connected with the interlacement location in the lockstitch link, as shaped by the ratio of the static forces put against the threads forming the stitch. The interlacement located inside the needle channel, at the specific level of the stitch link height, is carried out during the last stage of the machine working cycle, known as the stitch tightening. That process seems to be crucial for the analysis of the thread working conditions in the sewing machine, because it first determines the stitch structures, and secondly the dynamic force generated in the thread by the sewing process reaches the maximum value at that cycle stage, in most of the cases analysed.

According to research concerned with the thread working conditions in the sewing machine (including [5, 9 - 11]), factors may be specified which shape the stitch-tightening process as well as the qualitative relationships between them. In this paper, we present a model of the stitch tightening process, which expresses in a quantitative way the relationships between the sewing process’ technological parameters and the stitch tightening dynamic force. It also allows us to predict the interlacement location in the stitch link under given technological conditions.

While creating the model of the stitch tightening process in dynamic conditions, the following assumptions have been made:
1) The course of both the needle and the bobbin hook threads along the stitch formation zone is simplified while passing over some barriers, and consequently, a three-dimensional system is reduced to a two-dimensional one.
2) The system analysed is built of two threads: the first one (needle thread) inserts the second (bobbin hook thread) into the needle channel made in the fabric. The needle thread and the bobbin hook thread are of the same characteristics.
3) The stitch tightening process is considered with reference to the currently formed stitch link only. It is assumed that the interlacements created in the previous machine working cycles occupy their final location in the seam.
4) The main sources of resistance while inserting the lockstitch interlacement into the needle channel are the friction of threads in the interlacement and the friction of the interlacement inside the needle channel. The mutual friction of both threads in the interlacement is described by Euler’s law, for which the friction coefficient is determined empirically. The friction resistances inside the needle channel are characterised by the maximum value of the force, as determined during the experimental research.
5) The friction of the needle thread in the thread take-up eye is considered as relatively small, in relation to the thread friction forces in the interlacement, and is passed over.
6) The deformation of the needle thread during the stitch tightening process has a resilient character, and may be described by Hooke’s law. Simultaneously, the relatively small length of the bobbin hook thread section which participates in the process analysed justifies being omitted from the considerations of the elongation of the above-mentioned thread.
7) The total mass of the thread sections taking part in the stitch tightening process is concentrated in the stitch interlacement. Additionally, the mass of bobbin case filled out by the thread is taken into consideration, and the following condition is fulfilled: the mass of bobbin case is several times greater than the mass of interlacement.
For the model of the stitch tightening process which we worked out, according to Newton’s second law of dynamics, the following equation may be formulated with reference to the thread mass \( M \) concentrated in the interlacement \( Q \) (Figure 1.b):

\[
M \ddot{s}(t) = -s_1 - s_2 - T + s_3 + s_4
\]

where the values of the forces in the individual thread sections \( s_1, \ldots, s_4 \) are unknown.

Such values may be determined by analysing the forces and force moments against the thread sections distinguished (Figure 1.b, c, d), and formulating adequate equations for the interlacement area, the thread take-up eye area, the tensioner area in case of the needle thread, the interlacement area and the bobbin case area for the bobbin hook thread.

In the case of the bobbin hook thread, it was assumed that unreeling from the bobbin case and inserting a new thread section into the stitch formation zone occur during the whole process of the stitch tightening. From the equations formulated with reference to the bobbin case rotation centre, the following relationships are obtained:

\[
s_1 = s_2 \exp(\mu \pi) \quad (2)
\]

\[
(s_2 - s_0) = l_z \phi''(t) \quad (3)
\]

Simultaneously, the equation of the bobbin hook thread length balance in the stitch formation zone may be written as:

\[
l_1 + l_2 + \phi(t) R = l_1 + l_2 + 2x(t) \quad (4)
\]

and after simplification and two-time differentiation, the following relationship is obtained:

\[
\phi''(t) = 2 R \cdot x(t) \quad (5)
\]

The inertia moment of the bobbin case, determined in relation to the horizontal axis of its rotation, is expressed by the relationship:

\[
l_z = \frac{m_c R^2}{2} \quad (6)
\]

Thus, equation (3) may be written as follows:

\[
s_2 = s_0 + m_i \phi''(t) \quad (7)
\]

With reference to the needle thread in the model devised, the process of the location of interlacement in the needle channel has been divided into two stages. The first is characterised by elastic stretching of the needle thread; such a stretching is considered as being proportional to the geometrical load (displacement) magnitude \( u(t) \), by simultaneously blocking the needle thread end in the tensioner (stage I). The second stage is connected with inserting a new needle thread section \( k \) into the stitch formation zone (stage II). At this stage of the stitch tightening process, feeding the stitch formation zone by the needle thread section from the tensioner is assumed to be the only source of the needle thread’s increase in length.

For stage I of the stitch tightening process, the following equations with reference to the individual needle thread sections may be presented:

\[
s_3 = \frac{s_1}{\exp(\mu \pi)} \quad (8)
\]

\[
s_5 = s_4 \quad (9)
\]

According to the assumption made at the beginning, the needle thread undergoes deformation under the influence of the outer forces put against the thread in the stitch tightening process. Thus, for each individual section, as distinguished conventionally in the needle thread, i.e. \( l_3, l_4, l_5 \), the relationships in the following form may be formulated:

\[
l_i = l_i(1 + e_i) \quad (10)
\]

The deformations of the needle thread in the stitch tightening process are elastic, and subordinate to Hooke’s law. Therefore, the relationships may be written as follows:

\[
e_i = \frac{e_i}{\exp(\mu \pi)} \quad (12)
\]

\[
e_i = e_i \quad (13)
\]

Simultaneously, in the case of the needle thread, the following equations of this

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**Figure 1.** Physical model of the dynamic placing of the lockstitch interlacement inside the needle channel; a) general view; the forces and force moments acting against the thread sections are presented in the: b) interlacement area, c) thread take-up eye area, d) bobbin hook area.
thread balance in the stitch formation zone (corresponding with stage 1 of the stitch tightening process) may be formulated:

\[ L = L_1 + L_2 + L_3 \]  
\[ l_t^i = a + l_t^i \]  
\[ l_t^i + l_t^i + L_t^i = L_i + 2u(t) - x(t) \]  
\[ l_t^i = l_t^i = a - b \]

Equations (15) to (17), after taking the general relationship (10) into consideration, can be written as:

\[ l_t^i = a + l_t^i (1 + e_i) - x(t) \]  
\[ l_t^i = a + l_t^i (1 + e_i) + l_t^i (1 + e_i) = L_t + 2u(t) - 2x(t) \]  
\[ l_t^i (1 + e_i) - l_t^i (1 + e_i) = a - b \]

Solving the system of equations (12) to (14) and (18) to (20) with respect to the unknowns: \( l_t^i, l_t^i, l_t^i, e_i, e_i, e_i \) permits the following relationship to be formulated:

\[ \Delta = E \Delta_A \]  

where the absolute elongation \( \Delta \) is described by the function (22).

Providing the determined forces \( s_t, ... \), \( s_t \) into relationship (1), equation (23) is obtained. The geometrical loading \( u(t) \), as generated by the thread take-up, is defined by the time function as follows:

\[ u(t) = c_1 t^2 + c_2 t \]

After taking the above-mentioned relationship into consideration, equation (23) takes its final form in equation (25).

The differential equation obtained presents a function of variables \( t \) and \( x(t) \). If there are no algebraic solutions, the value of the variable \( x(t) \) to be searched permits us to calculate the value of the absolute interlacement placing index \( x(t) \) increases in a non-linear way by the increase of the argument \( t \). The interpolating function obtained permits us to calculate the value of the interlacement placing index \( x(t) \).

\[ x(t) = \frac{1}{2(M + m_e + \Delta e_i)} \left( e^{e_i} (e^{e_i} s_t^2 - e^{e_i} s_t^2 + s_t^2 + e^{e_i} s_t^2 - e^{e_i} u^2 T + \right. \]
\[ \left. +2e^{e_i} s_t^2 + 2e^{e_i} s_t^2 + 2e^{e_i} s_t^2 + 2e^{e_i} s_t^2 + 2e^{e_i} s_t^2 + 2e^{e_i} s_t^2 + 2e^{e_i} s_t^2 + 2e^{e_i} s_t^2 + 2e^{e_i} s_t^2 + 2e^{e_i} s_t^2 \right)


Table 1. Set of the model parameter values.

<table>
<thead>
<tr>
<th>Description of the parameter</th>
<th>designation, unit</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>total length of the needle thread in the stitch formation zone</td>
<td>( L_1 ), m</td>
<td>0.3</td>
</tr>
<tr>
<td>distance between the lower plane of the sewn material and the seizing point of the thread in the tensioner</td>
<td>a, m</td>
<td>0.2</td>
</tr>
<tr>
<td>distance between the lower plane of the sewn material and the seizing point of the thread in the last interlacement</td>
<td>b, m</td>
<td>0.006</td>
</tr>
<tr>
<td>half of the thread take-up eye acceleration</td>
<td>( c_1 ), m/s²</td>
<td>-14.8</td>
</tr>
<tr>
<td>speed of the thread take-up eye</td>
<td>( c_2 ), m/s</td>
<td>3.5</td>
</tr>
<tr>
<td>bobbin case radius</td>
<td>R, m</td>
<td>0.009</td>
</tr>
<tr>
<td>mass of the bobbin case full of the thread</td>
<td>( m_b ), kg</td>
<td>0.006</td>
</tr>
<tr>
<td>diameter of the sewing thread</td>
<td>d, m</td>
<td>0.0002</td>
</tr>
<tr>
<td>dynamic module of the thread initial rigidity</td>
<td>( E ), N/m²</td>
<td>5 10⁸</td>
</tr>
<tr>
<td>thread mass gathered in the interlacement</td>
<td>M, kg</td>
<td>0.001</td>
</tr>
<tr>
<td>friction coefficient of the thread inside the interlacement</td>
<td>( \mu ), -</td>
<td>0.4</td>
</tr>
<tr>
<td>maximum friction force of the interlacement in the needle channel</td>
<td>( T ), N</td>
<td>0.3</td>
</tr>
<tr>
<td>restrain force of the bobbin hook thread</td>
<td>( s_b ), N</td>
<td>0.2</td>
</tr>
<tr>
<td>restrain force of the needle thread (put in the tensioner)</td>
<td>( P ), N</td>
<td>3.5</td>
</tr>
<tr>
<td>length of stitch</td>
<td>s, m</td>
<td>0.0025</td>
</tr>
<tr>
<td>height of material package</td>
<td>h, m</td>
<td>0.002</td>
</tr>
</tbody>
</table>
within the established integration limits of the argument. According to the assumptions for stage I of the stitch tightening process, at the moment \( t = 0.0026 \) s the interlacement will occur at the location \( x(t) = 0.347 \times 10^{-3} \) m, providing that no new section of the thread will be inserted from the tensioner into the stitch formation zone, and so stage II of the stitch tightening will not ensue.

The model presented above expresses the correct description of the stitch tightening when the value of the force \( s_d \) which is rising in the thread, is lower than the force of thread restraint in the tensioner (i.e. until time \( t_f \)). Next, the appearance of the insertion of the new thread section \( k \) into the stitch formation zone should thus be taken into consideration.

The relationship (3) presents the basis for formulating the differential equation for stage 2 of the stitch tightening process. The relationships (4), (5), (6) and (23) also remain correct. As a consequence, the following equation may be written down:

\[
x'(t) = -\frac{s_0 - e^{\alpha s} x_0 + s_z e^{\alpha s} x - \frac{T}{M + m_c + e^{\alpha s} m_c}}{\Delta \sigma}
\]

(27)

where the value of the force \( s_d \) (Figure 1.b) is equal to the value of the thread restraint force in the tensioner \( P \):

\[
s_d = P
\]

(28)

The differential equation obtained gives the following algebraic solution after integration, which permits us to calculate the final interlacement location in the needle channel at the final moment of the stitch tightening process (29), where the values: \( x_p = x(t_f) \) and \( V_p = x'(t_f) \) are the border conditions for equation (27).

In the technological conditions characterised in Table 1, the value of the force \( s_d \) will reach a level equal to the assigned restraint force \( P \) after the time \( t_f = 1.24 \times 10^{-3} \) s, for which moment \( x_p = 2.26 \times 10^{-5} \) m and \( V_p = 8.29 \times 10^{-2} \) m/s. After providing the calculated border values into the relationship (29), the quadratic equation for the interlacement location as the function of time \( x(t) \) is obtained:

\[
x(t) = 5.067(1.967 \times 10^{-6} + 0.0123t + 11.571/t^2)
\]

(30)

which is presented graphically in Figure 3.

Under the given technological conditions, the final location reached by the interlacement in the needle channel after the stitch tightening process is described by the value \( x(0.0026) = 0.244 \times 10^{-3} \) m. After relating the interlacement location calculated to the total height of the material package, the following value of the interlacement location index will be obtained as \( u = 0.122 \).

In the second part of this paper, we will present the verification of the stitch tightening model devised by means of the experimental research conducted in the sewing conditions.

References

2. Więziak W., Technological assessment of sewing machines regarding stitch formation zone research (in Polish), Odzież [XXV], 1974, nr 3, s.54-57.