Computational Modelling of Pollutant Dispersion in a Cotton Spinning Mill

Introduction

In the last few years, considerable efforts have been made towards the definition of new ventilation standards in the design of buildings. The standards are presented e.g. in [4, 8 - 10]. Except for the simplified single-zone and multi-zone models [2, 11, 15, 16], nowadays sophisticated, professional models of computer fluid dynamics (CFD) e.g. VORTEX, STAR-CD, FLUENT can be applied to air flow analysis. The finite element method (FEM) [1] and finite volume method (FVM) [6, 7] are mostly applied. The aim of these calculations is to determine the air speed field inside a building [13, 17, 19]. The ventilation system and air flow influence the dispersion of pollutants, as well as the thermal environment and air quality inside buildings [5].

Textile plants are among the most polluting industrial buildings. The special conditions caused by manufacturing processes influence both the quality of materials and human health. Extremely hazardous conditions are commonly found in cotton spinning mills. A lot of dust particles appear in the initial phases of the cotton yarn manufacturing process, mainly in carding, drawing and roving. Hence, the design of the ventilation system in this kind of manufacture is of particular importance. This paper presents the author’s own models and computer programme that allows us to describe the flow of air in buildings with simple geometry. These models can be applied to the numerical analysis of the pollutant dispersion in a cotton mill, which is the subject of the second part of this work.

Transport equations

The basic equations describing the flow of fluid and heat within a building, as well as a k-ε (two-equation) model of turbulence, are partial differential equations (PDEs), which have the form presented below in the Cartesian coordinate system [6, 7].

Conservation of mass (continuity equation)

\[ \sum_{i=1}^{3} \frac{\partial U^{(i)}}{\partial x^{(i)}} = \frac{\partial}{\partial t} \frac{\rho}{c_{p}} = 0, \quad (1) \]

where: \( U \) is the vector of wind speed, \( x^{(i)} \) are the coordinates of a point in the Cartesian coordinate system.

Conservation of momentum (Navier-Stokes equations)

\[ \rho \frac{\partial U^{(i)}}{\partial t} + \rho \frac{\partial U^{(i)}}{\partial x^{(i)}} = - \frac{1}{\rho} \frac{\partial P}{\partial x^{(i)}} - \frac{\mu}{\rho} \frac{\partial U^{(i)}}{\partial x^{(i)}} - \frac{\mu}{\rho} \left( \frac{\partial U^{(j)}}{\partial x^{(i)}} + \frac{\partial U^{(i)}}{\partial x^{(j)}} \right) \frac{\partial U^{(j)}}{\partial x^{(j)}} + \rho g \delta^{(i)} \quad (2) \]

Conservation of energy (heat transfer)

\[ \rho c_{p} \frac{\partial T}{\partial t} + \rho c_{p} \frac{\partial U^{(i)}}{\partial x^{(i)}} = \frac{1}{\rho} \frac{\partial P}{\partial x^{(i)}} - \frac{\mu}{\rho} \frac{\partial U^{(i)}}{\partial x^{(i)}} - \frac{\mu}{\rho} \left( \frac{\partial U^{(j)}}{\partial x^{(i)}} + \frac{\partial U^{(i)}}{\partial x^{(j)}} \right) \frac{\partial U^{(j)}}{\partial x^{(j)}} - \frac{\mu}{\rho} \frac{\partial U^{(i)}}{\partial x^{(i)}} \frac{\partial U^{(j)}}{\partial x^{(j)}} - \frac{\mu}{\rho} \frac{\partial U^{(i)}}{\partial x^{(i)}} \frac{\partial U^{(j)}}{\partial x^{(j)}} + \rho \frac{\partial T}{\partial x^{(i)}} \frac{\partial U^{(i)}}{\partial x^{(i)}} + \rho \frac{\partial T}{\partial x^{(i)}} \frac{\partial U^{(i)}}{\partial x^{(i)}} \quad (3) \]

where: \( \rho \) is the fluid density, \( \mu \) is the dynamic viscosity of fluid, \( \mu \) is the coefficient of turbulent viscosity. \( \mu \) is the turbulent viscosity, \( \mu_T \) is the eddy viscosity, and \( c_p \) is the heat capacity at constant pressure.

Equations: 2, 3, 4 and 5. The Navier-Stokes equations, the thermal energy, the transport, and the kinetic energy dissipation equations.
is the thermal conductivity. Subintervals may be of equal size, are denoted as \( i \), \( j \), \( k \) – this system of algebraic and ordinary differential equations is solved by reducing the task to the problem of solving systems of algebraic equations (linear or non-linear).

Normally one of the following methods is used to discretise the equations:
- finite difference method (FDM),
- finite element method (FEM),
- finite volume method (FVM).

Because of the relatively easy interpretation of boundary conditions as well as the numerical effectiveness [6, 18], the finite volume method is applied in this paper.

The second stage of solving the system of ordinary differential equations with respect to \( t \) normally uses one of the following methods:
- explicit scheme,
- implicit scheme,
- Crank-Nicholson scheme [12].

\( c_U \) is the empirical constant, equal to 0.09 [12],
\( K \) is the kinetic energy of turbulence,
\( \varepsilon \) is the kinetic energy dissipation rate, \( g^{(r)} \) is the body forces in the \( r \) direction, \( p = p(t, x^{(1)}, x^{(2)}, x^{(3)}) \) is the pressure, \( \delta_{ij} \) is Kronecker’s delta.

Transport equation for \( K \) is presented as equation (5) where:
- \( K \) is the turbulent conductivity coefficient,
- \( \lambda_T = \mu_T \sigma_c / \rho c_p T \) is the turbulent conductivity coefficient,
- \( \sigma_c \) is the specific heat coefficient,
- \( \rho c_p \) is the Prandtl number,
- \( \sigma \) is the intensity of internal heat sources.

Kinetic energy dissipation is presented in equation (3) where:
- \( c_\varepsilon \) and \( c_\varepsilon x \) are empirical constants, equal to 1.44, 1.92, 1.03, respectively [12].

Solution of Equations 1 - 5 normally proceeds as follows:

**Stage 1** – the equations are transformed into a system of algebraic and ordinary differential equations with respect to variable \( t \). This stage, which eliminates derivatives with respect to spatial variables \( x^{(1)}, x^{(2)}, x^{(3)} \), is called the stage of discretising equations.

**Stage 2** – this system of algebraic and differential equations is solved by reducing the task to the problem of solving systems of algebraic equations (linear or non-linear).

In order to ensure a high degree of accuracy and stability of numerical solutions, we will use the Crank-Nicholson method. At this stage there are usually problems with the iterative solving of equations after discretisation (which is connected with approximation of the field of pressures) and solving large systems of algebraic linear equations.

Both procedural stages are discussed below, especially that which takes into account boundary conditions relating to the problem of determining components of the speed vector \( U = \{ U^{(1)} U^{(2)} U^{(3)} \}^T \) in a closed domain.

### Numerical solving of PDEs

The domain has the form of a rectangular prism, shown in Figure 1. Air motion is created by a number of inlets in which the speed of air in the direction normal to the surface is defined, as well as the outlets ensuring the exchange of air.

Within the domain an element is distinguished whose geometrical centre has the coordinates \( x^{(1)}, x^{(2)}, x^{(3)} \).

Before defining its dimensions, we shall describe the general method of denoting points and elements.

If we investigate axis \( r \), then, assuming that the length of the side of the domain in the direction \( r \) is \( L^{(r)} \), the denotation presented in Figure 2 is used.

Interval \( (0, L^{(r)}) \) is divided into \( n_r \) subintervals with lengths \( \Delta x^{(r)} = L^{(r)} / n_r \). In the centre of the subintervals points \( x^{(r)} = x^{(r)}_n + \sum_{i=1}^{n_r} \Delta x^{(r)} \) are located. The ends of the subinterval, whose centre is \( x^{(r)} \), are denoted as \( x^{(r)}_0 \) and \( x^{(r)}_n \). Subintervals may be of equal or different length.

The remaining equations, (1), (3), (4) and (5), are integrated over the volumes:

\[
V_{i,j,k} = \int_{x_{i1}^{(1)}}^{x_{i2}^{(1)}} \int_{x_{j1}^{(2)}}^{x_{j2}^{(2)}} \int_{x_{k1}^{(3)}}^{x_{k2}^{(3)}} dx^{(1)} dx^{(2)} dx^{(3)} = \Delta x^{(1)} \Delta x^{(2)} \Delta x^{(3)}
\]

for \( i = 1, ..., n_1, j = 1, ..., n_2, k = 1, ..., n_3 \).

In further considerations it is assumed that the boundary condition is one of the three following types.

**Condition type W (Wall)**

Viscosity means that at the points where the air is in contact with a wall, the following conditions apply (no-slip conditions)

\[
U^{(1)} = 0, \quad U^{(2)} = 0, \quad U^{(3)} = 0
\]
Condition type I (Inlet)

Here it is assumed that the speed of air flowing in or out (in a direction normal to the wall) is known, and the other two components of speed are equal to zero. This means that in this case one of the following possibilities occurs:

\[ U^{(1)} = U^{(1)}(t), \quad U^{(2)} = 0, \quad U^{(3)} = 0 \]  
\[ U^{(2)} = U^{(2)}(t), \quad U^{(1)} = 0, \quad U^{(3)} = 0 \]  
\[ U^{(3)} = U^{(3)}(t), \quad U^{(1)} = 0, \quad U^{(2)} = 0 \]  

The aspects of conditions (7) used depends on which direction the wall is perpendicular to.

Condition type O (Outlet)

It is assumed that through the outlet in the wall, air flows in or out. In this case, it is generally assumed that

\[ \frac{\partial U^{(k)}}{\partial x_i} = 0, \quad U^{(k)} = 0 \quad \text{for} \quad k \neq m, \]  
\[ p_{out} = 0, \]  

where:

- \( m = 1, 2, 3 \) (according to the location of the boundary),
- \( p_{out} \) - pressure outside the domain (usually assumed to be equal to zero).

For outlet conditions with the outlet type, we formulate additional simplified differential equations which are solved together with other equations. Having used formulae given in detail in [6, 7], the equations for the problem after the discretisation step can be written in the form of

\[ \text{Navier-Stokes, energy and } k-e \, \text{equations:} \]

\[ U + g_f(U,T,K,E) + A \cdot p = 0, \quad (9.1) \]
\[ T + g_f(U,T,K,E) = 0, \quad (9.2) \]
\[ K + g_f(U,T,K,E) = 0, \quad (9.3) \]
\[ E + g_f(U,T,K,E) = 0, \quad (9.4) \]

Continuity equation:

\[ B \cdot U = h, \quad (9.5) \]

where:

- \( A, B \) are matrices with constant coefficients, and
- \( U, p, T, K, E \) are vectors of unknowns in the nodes described above.

The difficulty of solving Equations (9) is due to:

1. the non-linearity of Equation (9.1),
2. vector \( p \) is not known while solving Equation (9.1).

In this paper we use a procedure which enables Equations (9.1) to be integrated according to the Crank-Nicholson scheme. In order to determine vector \( p \), we use an iterative procedure, which is a modification of the PISO algorithm [21, 22].

In order to perform the calculations, a computer program, called \textit{FV-mod}, was written in Object Pascal (Delphi v. 7.0). This gave us an effective code, and hence calculations could be performed on an IBM PC. By using dynamic matrices, it is possible to perform calculations for several thousand elements. The program includes:

- interactive input of data for calculations,
- for number of elements in \( x \)-direction, \( m = 2, \ldots, 15 \), \( \Delta x_{1,2,3} = 2 \, \text{m}, \Delta x_{4,5,6} = 0.5 \, \text{m} \).

\[ \text{Table 1. Estimated data based on the real parameters of the spinning mill building acc. to Figure 3.} \]

<table>
<thead>
<tr>
<th>Chamber</th>
<th>Efficiency of ventilators, m/s</th>
<th>Number of inlets</th>
<th>Total area, m²</th>
<th>Air speed ( U^{(3)} ), m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1, A2</td>
<td>25.0</td>
<td>8 (1 * 16 b)</td>
<td>16</td>
<td>-1.56</td>
</tr>
<tr>
<td>B1, B2</td>
<td>34.4</td>
<td>12 (9 * 16 b)</td>
<td>24</td>
<td>-1.44</td>
</tr>
<tr>
<td>C1, C2</td>
<td>52.2</td>
<td>12 (9 * 16 b)</td>
<td>24</td>
<td>-2.18</td>
</tr>
<tr>
<td>C3, C4</td>
<td>52.2</td>
<td>12 (9 * 16 b)</td>
<td>24</td>
<td>-2.18</td>
</tr>
<tr>
<td>D1, D2</td>
<td>34.4</td>
<td>12 (9 * 16 b)</td>
<td>24</td>
<td>-1.44</td>
</tr>
<tr>
<td>E1, E2</td>
<td>55.0</td>
<td>8 (9 * 16 b)</td>
<td>16</td>
<td>-3.44</td>
</tr>
</tbody>
</table>
Air flow in a cotton spinning mill

The procedures described in previous sections and the computer program are used in order to carry out calculations, as a result of which the distribution of the air speed vector in a cotton spinning mill is obtained. The main geometrical parameters and designations are given in Figure 3.

In Table 1 the channels are assigned to chambers and are given the parameters of air flux.

The domain was divided into 9750 control volumes with dimensions of 4 m in the $x^{(1)}$ direction and 1 m in the $x^{(3)}$ direction. The dimensions of elements in the $x^{(2)}$ direction change according to Figure 4.

In order to verify the program developed, the calculation results obtained by the FV-mod were compared with results from the Star-CD package.

With the Star-CD package, we used elements with dimensions of $4 \times 0.5 \times 1$ m, which resulted in a calculation model with 33750 elements.

Figure 5 shows differences between the FV-mod and Star-CD results. The percentage error is defined as follows:

$$
\Delta = \frac{U_{\text{STAR}} - U_{\text{FV}}}{U_{\text{max}}} \times 100\%
$$

where:

- $U_{\text{STAR}}$ - air speed obtained from STAR-CD,
- $U_{\text{FV}}$ - air speed obtained from FV-mod,
- $U_{\text{max}}$ - maximal value of air speed obtained from STAR-CD.

Figure 6 demonstrates differences in the calculation results for two values of $x^{(1)}$ ($38$ m) at a height of $1.5$ m.

Detailed analysis of results showed that the differences between values of the components of speed do not exceed $10\%$, thus we can state that the model and programs created to determine the field of air speed in a cotton spinning mill are correct. Figure 7 presents the results of calculations which show how the air speed changes in the domain. We can observe the significant air speed close to inlets and outlets.

Trash and dust in a cotton spinning mill

Trash and dust particles are “foreign” particles that are not related to any physical properties of cotton fibers; they are remains from the cotton plant and field and need to be extracted during the ginning and spinning process.

Trash is the general term used for larger impurities containing particles from the cotton plant itself and other plants (weeds) contaminating the cotton field. Dust describes smaller particles from the plant and simply dirt from the cotton field that sticks to the plant during harvesting.

Figure 8 shows the definition of trash and dust particles by size, as recommended by the ITMF (the International Textile Manufacturers Federation) [3].

Different parts of cotton contaminants are present in Figure 9 (see page 110). Foreign materials, such as plastic fragments, are crushed into fine fibres or fine segments during opening and cleaning.

Figure 7. Air speed in the domain, $x^{(3)}=1.5$ m.

Figure 8. ITMF definition of dust and trash particles [3].
Dispersion equation and its discretisation

The concentration of pollutants in a building depends on two basic factors: the intensity of emission sources and the effectiveness of the ventilation system used. In a cotton spinning mill the emission of pollutants is directly connected with the material applied to the manufacturing process, while the ventilation system mainly influences air flow in the domain considered. The previous sections of the paper present models and programs that allow us to determine the air field in the cotton spinning mill considered. Here we present models and calculation results connected with pollutant emission and dispersion.

The dispersion partial differential equation describing both the advection and diffusion of the pollutant can be written in the form - equation (11) [4, 5, 24], where:

\[ \phi(t, x^{(1)}, x^{(2)}, x^{(3)}) \]

is the concentration of the pollutant, \( \mu^{(1)} = \mu^{(2)} = \mu^{(3)} = \frac{1}{\rho} \frac{\mu_T}{S_c T} \) is the diffusion coefficient (in the case of \( K-\varepsilon \) turbulence model), \( S_c T \) is Schmidt’s turbulent number, \( I \) is the rate of pollutant emission (absorption) per unit volume.

According to the finite volume method, after the integration of elementary volumes, the equation is replaced by a system of ordinary differential equations, which can be written in the following form [6, 7]:

\[
\frac{d\phi}{dt} + A_1(\phi) + A_2(\phi) + A_3(\phi) = f, \quad (12)
\]

where:

\[ \phi = [\phi^{(1)}, \ldots, \phi^{(n)}, \ldots, \phi^{(n)}, \ldots, \phi^{(n)}] \]

is the vector of unknown values of function \( \phi \) at points of discretization, which has \( n = n^{(1)} \cdot n^{(2)} \cdot n^{(3)} \) components, \( A_1, A_2, A_3 \) are defined as follows:

\[ A_{1,i,j,k} = A_{1}(\phi^{(1)}_{i,j,k}), A_{2}(\phi^{(2)}_{i,j,k}), A_{3}(\phi^{(3)}_{i,j,k}) = f. \]

and by the carding process. After carding, the foreign material is often represented in the card sliver as a cluster of coloured fibres [23].

**Table 2. Concentration of dust and emission intensity around textile machines [13].**

<table>
<thead>
<tr>
<th>Number of source</th>
<th>Name of machines</th>
<th>Mean concentration, mg/m³</th>
<th>Emission intensity, mg/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Carding engines</td>
<td>2.45</td>
<td>0.1881</td>
</tr>
<tr>
<td>2</td>
<td>Drawing frames</td>
<td>2.92</td>
<td>0.0313</td>
</tr>
<tr>
<td>3</td>
<td>Lap former</td>
<td>2.97</td>
<td>0.1860</td>
</tr>
<tr>
<td>4</td>
<td>Combers</td>
<td>2.56</td>
<td>0.0080</td>
</tr>
<tr>
<td>5</td>
<td>Roving frames</td>
<td>3.13</td>
<td>0.5885</td>
</tr>
<tr>
<td>6</td>
<td>Spinning frames</td>
<td>1.32</td>
<td>0.2490</td>
</tr>
<tr>
<td>7</td>
<td>Twisters</td>
<td>1.30</td>
<td>0.4745</td>
</tr>
<tr>
<td>8</td>
<td>Winding machines</td>
<td>0.63</td>
<td>0.2683</td>
</tr>
</tbody>
</table>
Figure 10. Textile machines in the spinning mill under consideration: 1 – carding machine, 2 – drawing frame, 3 – lap former, 4 – comber, 5 – roving frame, 6 – spinning frame, 7 – twister, 8 – winder. Measurement points are indicated by S1 - S4.

bilisation scheme applied (UpWind and QUICK are available) [6, 7, 19].

This form of equation allows the fractional step method to be applied with respect to successive variables [18]. Brzozowska L. and Brzozowski K. [6] present the results of application of the one and two cycle schemes, as well as the Crank-Nicholson method.

Emission of pollutants in a cotton spinning mill

In the cotton spinning mill considered, the sources of dust are the textile machines used at typical stages of the yarn formation process. The dust emitted has different characteristics, consisting of sand, boils seed of cotton, parts of the stem, or small parts of yarn. The main parameters of the domain are presented in chapter “Air flow in a cotton spinning mill” (see Figure 4). The position of textile machines in the hall is presented in Figure 10.

Values of dust concentration and emission intensity obtained by measurements are presented in Table 2.

Calculation results

On the basis of the models and programs developed, a series of calculations were carried out. In the first stage the calculation results were compared with those obtained by measurements. In Table 3 (see page 112) we can see the mean dust concentration at points S1 - S4.

The differences between the measured and calculated concentration do not exceed 7% at points S1 - S4. Thus, we as-
sume that the computer program gives correct results.

The concentrations of dust obtained at a height of 1.0 m and 1.5 m by calculations are presented in Figure 11.

Figure 12 shows how the concentration of pollutants changes with height $x^{(3)}$.

The maximal dust concentration can be observed in the environs of rowing machines and twisters.

In order to find how the ventilation system applied influences the concentration of dust in the cotton spinning mill, the cases described in Table 4 were simulated.

The concentration of pollutants at height $x^{(3)} = 1.5$ m for the cases considered are presented in Figure 14.

In Figure 13 we can see how the air velocity changes in inlets for the cases considered.

Case C1 describes the existing ventilation system. Case C2 has a different profile of the inlet air speed. In case C3 the inlet speeds are 1.25 of the speeds in case C1, while in case C4 all the inlet speeds are equal to 3 m/s.

The concentration of pollutants at height $x^{(3)} = 1.5$ m for the cases considered are presented in Figure 14.
It can be observed that in case C2 the dust concentration is higher (especially close to emission sources and outlets O1-O5) than in the case of the existing ventilation system (case C1). In case C3, when the ventilator rate is 25% higher than in case C1, the dust concentration is 12 - 34% lower. Case C4 gives a concentration of pollutants about 20% lower than C1.

The analysis of calculation results proves that the existing ventilation system satisfies the requirements of the Polish Standard [20]. A significant reduction in dust concentration can be obtained only by increasing the capacity of the existing ventilation system.

### Summary

This paper presents models and programs allowing us to solve the problems of air flow and dispersion of pollutants in a cotton spinning mill. The finite volume method has been applied to discretise PDEs, which describe the transport of dust and its diffusion. The correctness of the models and programs were checked by comparison of the calculation results obtained with those acquired by application of the STAR-CD package or by measurements. An acceptable capability of results has been achieved. The calculations concern a real cotton spinning mill in the south of Poland.

The authors of the paper believe that the approach presented as well as the models and programs developed can be applied for the effective simulation of pollutant dispersion in the design process of other textile plant buildings.

### References


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