Transverse Vibration of Papermaking Felt

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Abstract

The papermaking felt of high-speed papermaking machine vibrates seriously in a transverse direction. In this study, the transverse vibration of papermaking felt was analysed mechanically with respect to flexural rigidity, and a model was developed. The analytical resolution is obtained by the separation variable method. The result shows that it is a sinusoidal steady-state response. The natural frequency is calculated by the principle of free vibration. The flexural rigidity and density of papermaking felt are the main factors influencing its natural frequency. The validity of the model is verified theoretically and experimentally. The amplitude of the transverse vibration was tested by an optical non-contact measuring system. The amplitude-frequency characteristic curve shows that the natural frequency measured agrees well with the theoretical one.

Key words: papermaking felt, flexural rigidity, transverse vibration, natural frequency, PSD.

Introduction

Papermaking felt is used for the conveyor belt and filtration belt in the press section of a papermaking machine. The transverse vibration of papermaking felt increases steeply during high-speed operation, which will interrupt production and even damage the machinery. Nowadays, growing attention is focused on the operational stability of papermaking felt.

The transverse vibration of papermaking felt is closely related to its characteristics and mechanical devices. Moreover, it is similar to the vibration of general textiles. Gligorijevic [1] studied yarn oscillation in warp knitting. The free oscillatory frequencies of yarns are related to the length of yarn sections and the tensioning force. Aubry et al [2] investigated the vibrating characteristics of textile surfaces under uniaxial tension. However, they did not consider the properties of yarn or fabric in their analysis. Matsudaira [3] discussed the bending vibration characteristics of fabrics and new mechanical parameters which represent this character. The study evaluated the wearing characteristics of fabrics, not operational stability. Papermaking felt is a special kind of fabric, different from yarn and general fabric. As regards paper machines and paper technology, more attention should be paid to the design of papermaking felt [4]. Papermaking felt is actually a mass-spring system [5]. Resonance can be avoided by designing the structure of papermaking felt [5 - 7]. It is also a damp material that reduces the amplitude of vibration and even attenuates the vibration to zero. However, the natural frequency of papermaking felt changes during its service life. The flexural rigidity of the papermaking felt used decreases due to severe wear [8], as a result of which the natural frequency of the papermaking felt decreases and gradually becomes closer to the natural frequency of the papermaking machine. The natural frequency of papermaking felt is considered in the designing of Tamfelt [9]. However, the designing and manufacturing of papermaking felt mainly depend on experience.

The transverse vibration of papermaking felt is studied mechanically in this paper. An identified model is used to simulate the transverse vibration of papermaking felt, in which the flexural rigidity is considered. The experiment designed verifies that the model is reliable.

Theoretical analysis of the papermaking felt system

Modelling of papermaking felt

The structure of the press section of a papermaking machine is shown in Figure 1. In order to analyse the transverse vibration of papermaking felt, the section...
designated as a broken circle is simplified as a sketch map in Figure 2.

Under uniaxial tension, the transverse vibration of papermaking felt is something like that of string. Papermaking felt has a unique base cloth, which makes its flexural rigidity greater than general fabrics. Therefore, the transverse vibration of papermaking felt is similar to that of a Euler beam.

Equation of motion

The governing differential equation of motion is given using Newton's second law, which is a combination of the string [10] and beam models [11].

\[ T \frac{\partial^2 y}{\partial x^2} - EI \frac{\partial^4 y}{\partial x^4} - m \frac{d^2 y}{dt^2} = 0 \]  

(1)

where:
- \( m \) – linear density of the sample in kg/m,
- \( T \) – tensioning force in N,
- \( EI \) – flexural rigidity of the felt sample in N·m².

The boundary conditions are as follows:

\[ y(0,t) = 0, \quad y'(0,t) = 0, \quad y(l,t) = A e^{j\omega t}, \quad y''(l,t) = 0 \]  

(2)

The separation variable method is used to solve Equation 1. Supposing \( y = e^{j\omega t} \), where \( \mu \) is the wave number, and \( \omega \) the angular frequency, the equation of motion can be obtained as follows:

\[ y = \left( a \cosh \left( \sqrt{p_1} x \right) + \right. \]
\[ + b \sinh \left( \sqrt{p_1} x \right) + c \cos \left( \sqrt{p_1} x \right) + d \sin \left( \sqrt{p_1} x \right) \right) e^{j\omega t} \]

(3)

where

\[ p_1 = \frac{-T + \sqrt{T^2 + 4m\omega^2 EI}}{2EI}, \]

\[ p_2 = \frac{-T - \sqrt{T^2 + 4m\omega^2 EI}}{2EI}. \]

Substituting the boundary condition in Equation 2 into the equation of motion in Equation 3 to obtain the integration constant, the equation of motion becomes

\[ y = \frac{A}{(p_1 + p_2)} \left( \frac{p_1 \sinh \left( \sqrt{p_1} x \right)}{\sinh \left( \sqrt{p_1} l \right)} + \right. \]
\[ + \left. \frac{p_2 \sin \left( \sqrt{p_1} x \right)}{\sin \left( \sqrt{p_1} l \right)} \right) e^{j\omega t} \]

(4)

The equation of motion in Equation 4 is of a standard wave form, which means that the behaviour of every point on the papermaking felt is simple harmonic vibration.

Natural frequency

Substituting the free boundary condition into Equation 3, we get Equation 5.

In order to obtain a non-zero solution of Equation 5, the determinant of the equation system should be zero. That is

\[ \sin \left( \sqrt{p_1} l \right) = 0. \]

The natural frequency of the papermaking felt is

\[ f = \frac{n}{2l} \sqrt{\frac{EI}{m}} \left( \frac{nx}{l} \right)^2 + \frac{T}{m} \]  

(6)

where \( l \) - the length of the sample in m.

From Equation 6 we can see that the natural frequency of papermaking felt is related to the tension \( T \), flexural rigidity \( EI \), linear density \( m \) and the length \( l \). For the papermaking machine, the tension \( T \) and length \( l \) are invariable. Therefore, the flexural rigidity \( EI \) and linear density \( m \) of papermaking felt are the main factors influencing its natural frequency. The structure of papermaking felt is a key parameter determining the flexural rigid-

![Figure 3. Diagram of experiment (a).and frequency domain curve (b).](image-url)
ity $EI$. Structure type and density design are the main aspects of papermaking felt. As long as the natural frequency of the papermaking felt is far from the running frequency of the contacting rollers, serious transverse vibration can be avoided.

### Natural frequency ($EI << T$)

\[
 p_{1,2} = \frac{T}{2EI} \left( \sqrt{\frac{T}{2EI}} \right) + \frac{m_0^3}{EI} = \frac{T}{2EI} \left( \sqrt{1 + \frac{4m_0^2EI}{T^2}} - 1 \right)
\]

If $m_0^2 << (T/2EI)^2$, that is $EI << T^2/4m_0^2$, using the progression expansion, we can obtain

\[
 \sqrt{1 + \frac{4m_0^2EI}{T^2}} = 1 + \frac{2m_0^2EI}{T^2} + \frac{1}{4} \left( \frac{4m_0^2EI^3}{T^6} \right)
\]

Hence

\[
 p_i = \frac{m_0^3}{T}, \quad |p_i| = \frac{T}{EI} + \frac{m_0^3}{T} \quad (8)
\]

Substituting boundary conditions into Equation 3, $f_0 = \frac{n}{2l} \sqrt{\frac{T}{m}}$ is obtained, which is the same as the natural frequency of classical string [10].

### Conclusions

Papernaking felt plays an important role in paper machines. The transverse vibration of papermaking felt running at high speed is complex, which is affected by its own properties and the mechanical device.

In this study, the transverse vibration of papermaking felt was analysed mechanically. Under uniaxial tension and with its unique base cloth structure, the mixture model of string and beam can correctly characterise the transverse vibration of papermaking felt.

The transverse vibration of papermaking felt was tested by an optic non-contact measurement system, the result of which shows that the experimental natural frequency is close to the theoretical natural frequency.

The natural frequency of papermaking felt can be designed by adjusting its structure and density. If its natural frequency is far from the running frequency of the contacting rollers, serious transverse vibration can be avoided.

### References