Modelling of the Temperature Field within Knitted Fur Fabrics

Abstract

Knitted fur fabrics can be analysed as a textile composite made of two layers (a plait layer and a pile layer). The basic geometrical parameter is the relative cover i.e. the surface fraction of fibres, determined according to a computer technique of image processing and introduced into the homogenisation procedure. The heat transfer problem can be formulated within each layer by a second-order differential state equation and a set of boundary and initial conditions. State equations are solved numerically and the results visualised by the ADINA program. A simple numerical example is considered.

Key words: knitted fur fabrics, heat transport, temperature distribution.

Introduction

Knitted fur fabrics are applied in different branches of industry on the basis of their specific structure and properties. Fur fabrics are produced as industrial fabric and applied, for example, in toys and decorations. The main field of application is the clothing and footwear industry (furs, jackets, the thermal insulation of clothing etc.), and the basic application criteria are the thermal insulation of the textile product and the aesthetic appearance. The literature analysed does not discuss the following: the transport processes of energy and mass, the modelling of heat transfer and the wide spectrum of optimisation problems within knitted fur fabrics. The quality features of natural and knitted furs as well as the different problems of heat parameters were presented by Korliński [2,13]. Some problems concerning heat resistance and parameters describing the heat transfer were discussed by Korycki and Wiezowska [6].

Application, modelling and shape optimisation during the heat transfer as well as the problems connected have been widely discussed for conventional fabrics. Modelling the heat energy transfer within knitted fur fabrics is very difficult because it is necessary to determine the large number of phenomena (for example, the surface fraction of fibres within the fabric, the homogenisation of the structure, the contact of hairs within the pile layer etc.).

A typical knitted fur fabric has a non-homogeneous structure, each layer of which has an irregular random structure. To model the heat transfer, each layer of the complex structure (plait layer, pile layer) should be first homogenised. The homogenisation of textile structures is a complex and difficult process that can be solved by means of different methods. Tomeczek [10] introduced a fibrous reinforcement situated regularly within the filling of the composite material. Golanowski, Terada and Kikuchi [1] introduced different models of thermal homogenisation, i.e. the rule of mixture and the hydstatic analogy. Some authors describe the particular homogenisation of composite materials for unidirectional filling fibres, cf. Rocha and Cruz [10]. Liang and Qu [7] determined an equivalent diffusion coefficient for materials of considerable temperature difference between the two parallel external surfaces and the internal radiation within porous material. The basic geometrical parameters should be determined by means of experimental methods. Some authors have analysed an image of the surface of knitted fur fabrics. The best reference here can be Mikolajczyk [8] and the technique of image processing he introduced.

The main idea of the present paper is to model the heat energy transport of a textile composite of knitted fur fabric as well as solve the problem thereof. Mathematically speaking, we have a second-order differential equation accompanied by a set of boundary and initial conditions. The problem can be solved numerically and the results visualised by means of any graphics program as the temperature distribution within the structure. This class of problem has not yet been considered in the literature analysed concerning heat transport within knitted fur fabrics.

Structural parameters of knitted fur fabrics

Typical knitted fur fabrics have a 3D textile composite structure, cf. Figure 1. The literature analysed does not contain parameters describing fur fabrics. Plain stitch, which is a part of the plait layer, is described in some publications [2, 3, 7] in respect of the structural parameters and thermal correlations concerning the structure. We now have a great number of parameters, and modelling by means of standard methods is difficult. Thus, it is necessary to introduce some simplifications.

- Knitted fur fabrics have a 3D space structure, which is a sandwich composite. For simplicity we assume there are two basic layers of fur fabrics i.e. the plait layer and the pile layer.
- Fur fabrics can contain a lot of raw products: yarns (one or two threads creating the plait layer), bands (a mixture of two or three kinds of fibres), foamed glue (one type of glue or a mixture of two kinds).
- The basic structural parameters are not described within the European Standards EN and International Standards ISO.
- The plait layer is covered by foamed glue. It is also impossible to determine certain geometrical and material parameters after the finishing process of knitted fur fabrics by means of the standard methods. For the plait layer these are the stitch length and the parameters of yarns: the material, and the number and twist of yarn.
- The technological parameters of the structure are difficult to describe because different finishing processes can be applied, for example the drafting or stabilising of fur fabrics, the sizing connected with the drafting, as well as the shearing and fancy twisting processes. The final fur fabric has different structural parameters from the raw fabrics immediately after the knitting process.

The basic problems of effectively describing the energy transfer within knitted fur fabrics are the lack of sources (i.e. standards, scientific papers, descriptions...
of tests etc.) and the great number of different tests. Thus, the parameters determined are necessary to solve the state equation with respect to both boundary and initial conditions.

The relative cover of knitted fur fabrics within the plait layer was analysed according to a computer technique of image processing. There is a non-destructive and objective method which can determine the correlation between the structure and parameters of the manufacturing process. In this study, we have determined the mass fraction of fibres for knitted fabrics but not for a particular layer. The most important parameter for materials management is the relative cover, i.e. the surface fraction of fibres within the plait layer. Thus, the most effective method is a computer technique of image processing to create a 3D picture of knitted fur fabrics. The picture is then transformed into a black and white bit map, but the important question is to determine the precise border between the object measured and the background, which is the effect of picture segmentation, cf. Korlinski, Perzyna [3]. The values of the relative cover obtained (i.e. the surface fraction of fibres within the plait layer) for typical fur fabrics are equal according to test result - from 84.94% to 95.21%.

### Homogenisation of knitted fur fabrics

The homogenisation of particular layers is a typical method for composite structures, but it is not used for textile materials. Knitted fur fabrics are inhomogeneous, multilayered and have a sandwich structure. The composite structure contains a plait layer (made of fibres, yarns and foamed glue between them) and a pile layer (made of fibres, air between filaments, and air within the free spaces between fibres). The homogenisation of knitted fur fabric creates the homogeneous, orthotropic structure of the effective thermal conductivity coefficients.

There are different homogenisation methods to obtain the heat conductivity coefficient. Porous-capillary-channel materials were discussed by Wawszczak [13]. The heat conductivity coefficient is a function of the material conductivity \( \lambda_{\text{mat}} \) and the air or some other gas or liquid conductivity within the void spaces \( \lambda_{\text{fil}} \). Wawszczak determined the heat conductivity coefficient in the direction parallel to channels \( \lambda_{\text{par}} \) and orthogonal to channels \( \lambda_{\text{ort}} \) in the form

\[
\frac{\lambda_{\text{par}}}{\lambda_{\text{mat}}} = 1 - \xi \left( 1 - \frac{\lambda_{\text{fil}}}{\lambda_{\text{mat}}} \right);
\]

\[
\frac{\lambda_{\text{ort}}}{\lambda_{\text{mat}}} = 1 - \xi + \frac{\xi}{\lambda_{\text{fil}} / \lambda_{\text{mat}}},
\]

where \( \xi \) denotes the porosity coefficient defined as the proportion of the filling volume to the volume of material.

Tomeczek [11] analysed the reinforcement fibres situated regularly within the filling material. The heat conductivity coefficient of the material is \( \lambda_{\text{mat}} \) and that of the filling - \( \lambda_{\text{fil}} \). Let us assume a cylindrical shape of the reinforcement fibres of radius \( r \), which can be defined as the volume contribution within the material. The equivalent heat conductivity coefficient is equal to:

\[
\lambda = \lambda_{\text{fil}} \left( 1 - r^2 \right) + \frac{\lambda_{\text{mat}} \lambda_{\text{fil}}}{2 \lambda_{\text{mat}} \left( 1 - r^2 \right) + \lambda_{\text{fil}} r^2} r^2
\]

(2)

Golański, Terada and Kikuchi [1] introduced the classical ‘rule of mixture’, in which the equivalent heat conductivity coefficient has the form

\[
\lambda = \frac{V_{\text{mat}}}{V_{\text{mat}} + V_{\text{fil}}} \lambda_{\text{mat}} + \frac{V_{\text{fil}}}{V_{\text{mat}} + V_{\text{fil}}} \lambda_{\text{fil}}
\]

where \( \xi_{\text{mat}} \) and \( \xi_{\text{fil}} \) are the volume coefficients of the textile material \( V_{\text{mat}} \) and interfibre spaces \( V_{\text{fil}} \), respectively. In this study, this method was implemented into the solution procedure for the heat transfer problem.

The same authors define Turner’s model according to the hydrostatic analogy. The equivalent heat conductivity coefficient can be expressed as follows

\[
\lambda = \frac{V_{\text{mat}}}{V_{\text{mat}} + V_{\text{fil}}} \frac{\lambda_{\text{mat}} K_{\text{mat}}}{K_{\text{mat}} + K_{\text{fil}}} + \frac{V_{\text{fil}}}{V_{\text{mat}} + V_{\text{fil}}} \frac{\lambda_{\text{fil}} K_{\text{fil}}}{K_{\text{mat}} + K_{\text{fil}}}
\]

(4)

where \( K_{\text{mat}} \) is the volumetric modulus of the material, and \( K_{\text{fil}} \) - the volumetric modulus of the filling.

### Heat transfer model within knitted fur fabrics

Generally speaking, the heat transfer within knitted fur fabrics is a 3D space problem. Let us assume that the shape and heat transfer conditions are the same throughout the structure. It follows that a 3D space problem can be reduced to a 2D plane problem, cf. Figure 2. In some cases the heat is transported from the skin to the surroundings. If the problem is symmetrical or the side surfaces of the structure are isolated, we solve in fact a 1D heat transfer problem.

The state variable is the temperature \( T \). A typical fur fabric does not contain heat sources within its composite structure. There is an additional thin layer of air between the fur fabric and the skin. Thus, the temperature secures an optional microclimate, and a Dirichlet condition exists on this boundary portion \( T_{\Gamma} \). On the side surfaces of fur fabrics, the heat flux...
density can be neglected, \( q_n = 0 \), because heat energy is transported from the skin to the surroundings. The structure is subjected to Neumann conditions on these side boundaries, \( \Gamma_q \). The external surface is the boundary portion \( \Gamma_C \) subjected to convective heat flux, i.e. a third-kind boundary condition is applied. This part of the external boundary is additionally subjected to thermal radiation characterised by the Stefan-Boltzmann constant and real body properties i.e. the panchromatic emissivity, according to Zarzycki [14]. The internal boundaries \( \Gamma_N \) are characterised by fourth-kind conditions, i.e. the heat flux densities are the same for the common parts of internal boundaries. The initial condition describes the distribution of state variable \( T \) within the area \( \Omega \) bounded by the external boundary \( \Gamma \). The state equation and set of boundary and initial conditions have the form described for the \( i \)-th layer, according to Korycki [5] (cf. Figure 3)

\[
\begin{aligned}
\text{div} q^{(i)} &= c^{(i)} \frac{\partial T^{(i)}}{\partial t} \quad \text{within } \Omega; \\
q^{(i)} &= A^{(i)} \cdot \nabla T^{(i)} + q^{(i)} \quad \text{within } \Omega, \\
q_n^{(i)}(x, t) &= q_n^{(i)}(x, t) \quad x \in \Gamma_N; \\
T^{(i)}(x, t) &= T^0(x, t) \quad x \in \Gamma_i; \\
q_n^{(i)}(x, t) &= 0 \quad x \in \Gamma_q; \\
q_{se}^{(i)}(x, t) &= \beta \left[ T(x, t) - T_{\infty} \right] \quad x \in \Gamma_C; \\
q_{fo}^{(i)}(x, t) &= \sigma \varepsilon T^{4(i)}(x, t) \quad x \in \Gamma_C; \\
\sum q_{se}^{(i)}(x, t) + q_{fo}^{(i)}(x, t) &= \mathbf{q}^{(i)}(x, t) \quad x \in \Gamma_C; \\
T^{(i)}(x, 0) &= T_0^{(i)} \quad x \in (\Omega \cup \Gamma); \quad i = 1, 2.
\end{aligned}
\]

where \( q \) is the vector of heat flux density, \( \mathbf{q} \) the vector of initial heat flux density, \( q_n = n \cdot q \) the vector of heat flux density normal to the surface defined by unit vector \( n \) normal to this surface, \( A \) the matrix of heat conduction coefficients within the material, \( c \) the heat capacity, \( T \) the temperature, \( t \) the real time, \( T^0 \) the prescribed value of temperature, \( h \) denotes the surface film conductance, \( T_{\infty} \) the temperature of the surroundings, \( \sigma \) the Stefan-Boltzmann constant, and \( \varepsilon \) is the panchromatic emissivity of the material.

The problem can be considerably simplified for the steady heat transfer, i.e. for the constant value of the state variable in time. The temperature distribution can be also determined by the simple differentiation of the state equation with respect to the design variable.

The solution of the transient heat transfer for multilayer structures is complicated, and the problem should be solved by means of numerical methods. The solution methodology is shown in Figure 4. Firstly the material parameters and set of boundary and initial conditions are defined for each layer within the structure. Then we introduce two material layers: (i) a plait layer made of textile material and foamed glue, (ii) a pile layer consisting of fibres and air within the free spaces. The analysis procedure is based on the Finite Element Net, and the cross-section of the textile composite is divided into a net of 9-nodal rectangular elements. The state equations accompanied by the set of conditions are solved within the nodes, the results of which are so-called nodal values of temperature. This element is characterised by means of shape functions which help to approximate the temperature proportionally to the distance to a particular node. In fact we obtain a set of state variables within the textile composite at a defined time step, and the problem can be visualised by means of any graphics program. We introduced the values obtained into the ADINA program in a Windows environment. The calculations should be repeated till the last time step is introduced.

Illustrative example

Let us consider the isotropic properties of the material during heat transfer; this means that in this case the matrix of the heat conduction coefficients has only one component, which is the heat conduction coefficient determined by the homogeni-

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**Figure 2.** Homogenisation of the structures of knitted fur fabrics; 1 - skin, 2 - knitted fur fabric, 3 - any cut of fur fabric.

**Figure 3.** Model of transient heat transfer within knitted fur fabrics.

**Figure 4.** Algorithm for determination of the temperature within knitted fur fabrics for the transient heat transfer.
Let us solve the problem for the simplest case of steady heat transfer: the time derivatives of the temperature are equal to zero, and the right-hand side of the state equation within Equations (5) vanishes. The structure is made of two layers, and the heat transfer is a 1D problem. The internal boundary of the composite is defined for coordinate \( x \) by the heat flux density \( q_n \) and temperature \( T \). Generally speaking, the heat transfer problem is defined by adapting Equations (5). Let us next assume, for simplicity, that the radiative heat transfer is negligible in this particular case.

The heat transfer problem for the first layer can be adopted from Equations (5) as follows:

\[
A^{(1)} T^{(1)};xx = \text{within } \Omega; \\
q^{(1)} = -A_1\varepsilon_1 T_n = q_n^w \quad x \in \Gamma_1; \\
T^{(1)} = T^0 \quad x \in \Gamma_1; \\
q^{(1)} = 0 \quad x \in \Gamma_0; \\
q^{(1)} = q_n^0; \\
\text{where } A_1 = \lambda_1 \varepsilon_1 	ext{ is the matrix of the heat conduction coefficients of the first layer, and } \varepsilon_1 \text{ is the porosity of this layer. The second layer can be characterised by the following correlations:}
\]

\[
A^{(2)} T^{(2)};xx = \text{within } \Omega; \\
q^{(2)} = -A_2\varepsilon_2 T_n = q_n^w \quad x \in \Gamma_2; \\
q^{(2)} = 0 \quad x \in \Gamma_0; \\
q^{(2)}(x,t) = h[T - T_\infty] \quad x \in \Gamma_C; \\
\text{where } A_2 = \lambda_2 \varepsilon_2 \text{ and } \varepsilon_2 \text{ are the adequate parameters for the second layer.}
\]

Integrating the state equations with respect to design variables and introducing the boundary conditions, we obtain the following correlations

\[
\begin{align*}
T - T^0 + \frac{q_n^{w}}{\lambda_1 \varepsilon_1} x &= 0 \quad \text{for } 0 \leq x \leq x_w; \\
T - T_n + h \left( T - T_\infty \right) (x - x_w) &= 0 \\
&\quad \text{for } x_w \leq x \leq L
\end{align*}
\]

where \( L \) is the thickness of the whole material, cf. Figure 5. We see at once that the temperature is a linear function of coordinate \( x \) for both layers. The characteristic is different for every layer and depends on the material parameters and boundary conditions.

However, it is necessary to solve a second more comprehensive example -defining the material parameters of knitted fur fabrics. First we should introduce matrices of the heat transfer coefficients, and a substitute matrix of the coefficients can be determined according to one of the methods previously discussed (cf. subclause 3). The internal plait layer, which is in contact with the skin, is made of polyester and has a matrix of the heat transfer coefficients equal to \( A = 0.21 \text{ W/(mK)} \), see Urbanczyk [12].

The side boundaries \( \Gamma_q \) are characterised by the heat flux densities \( q_n = 0 \). The external part of the structure is subjected to thermal convection and thermal radiation. We assume that the surrounding temperature is equal to \( T = T_\infty = 0 \text{ °C} \), the heat convection coefficient \( h = 5 \), and the panchromatic emissivity is equal to \( \varepsilon = 0.8 \) [14].

The fibres within the pile layer are made of polyester (8.5% of the volume of the pile layer) and acryl (41.5% of the total volume). The free spaces within the pile layer are filled by air (50% of the total volume) of the matrix of the heat transfer coefficient \( A = 0.025 \text{ W/(mK)} \). Assuming the above parameters for acryl equal to \( A = 0.051 \text{ W/(mK)} \), the matrix of the heat transfer coefficient according to the rule of mixture is equal to \( A = 0.051515 \text{ W/(mK)} \).
solution procedures which minimize the calculation time. The boundary
and initial conditions can be introduced in different forms which are
not always applicable for empirical methods. An additional advantage
is the temperature distribution determined within the cross-section of the
fabric, which is impossible to obtain in empirical tests.
3. The method introduced can be also applied for the optimal shape design
of knitted fur fabrics with respect to maximal thermal insulation. Of
course, the problem needs additional practical verification, which is be-
yond the scope of the present paper.

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