Method of Separating Fabric from a Stack – Part 2

Kinga Stasik
Lodz University of Technology,
Department of Mechanics of Textile Machines
ul. Żeromskiego 116, 90-924 Łódź, Poland
E-mail: kinga.stasik@p.lodz.pl

Abstract
A mathematical model of the gripper mechanism with an electric drive, characterised by an increased work surface is presented in the paper. Equations describing the kinematic and dynamic characteristics have been formulated for this model. The resulting equations have been solved numerically. The calculation results illustrating the rheological reaction of the material to the operation of the gripper have been shown in diagrams.

Key words: gripper for textiles, computer control, mathematical model, garment production, robotisation.

Introduction. Mathematical models of grippers for textiles

Existing grippers very often use springs for compression [1 – 3]. Textiles are exposed to destruction because come into contact with the jaws in a very small area and they do not have the ability to control the compression force. Papers [4, 5] proposed an improvement associated with it, where the spring is replaced with a motor which is intended to prevent the crossing of the permissible pressures for the fabric so that there is no destruction.

Despite the use of the motor instead of the spring and the ability to control the size of the pressure on the material, forces acting pointwise on textiles may still be too large.

Reducing pressures necessary to maintain the fabric is possible by increasing the contact surface of the jaws. Therefore the solution to this problem proposed is a gripper for textiles with parallel fingers [5]. The fingers are in contact with the separating fabric over a much larger area than grippers gripping the fabric in a “pinch” [4].

The grippers discussed could serve as such for textiles, but they need to have the fabric between their fingers. Effective separation of the fabric requires a mechanism introducing the fabric between its fingers. This paper describes a mechanism for introducing the fabric between fingers using frictional forces, which can be realized by using an endless belt, pulling the fabric by force of friction in the direction of the blockage place, where is a fold formed entering between the belt and foot. The further movement of the belt will unfold the fabric between the elements involved in separating the fabric layer [6].

Equations of motion

Employing the principle of virtual works for moving parts of the gripper described in [6] in Figure 1 (see page 80), supplemented with moments of inertia of the guide rollers of the belt I_{2c}, I_{1c}, I_{dc} which move with velocity \( \omega \), the frictional forces between the belt and separating fabric layer \( T_1 \) and \( T_2 \) and the weight of the separating fabric \( m_g \) and \( m_v \), we can write a dynamic equation of motion in the following form:

\[
M_v \frac{d^2 \theta_v}{dt^2} = I_2 \frac{d \phi_2}{dt} + m_g \frac{d \phi_v}{dt} - m_v \frac{d \phi_v}{dt} - (T_1 + T_2) \frac{d \phi_v}{dt} = 0
\]

The forces of friction between the belt and separation fabric layer equal, respectively, to:

\[
T_1 = \mu_p (N_v + m_v g), \quad T_2 = \mu_p N_h
\]

Equation 1, taking account of the friction forces \( T_1 \) and \( T_2 \) and masses \( m_j \) and \( m_v \) (4a, 4b), as described in [6], after the transformation can be written as:

\[
I_2 \frac{d \phi_v}{dt} + I_1 \frac{d \phi_v}{dt} + (m_v + m_g) \frac{d \phi_v}{dt} + m_v \frac{d \phi_v}{dt} + (m_v + m_g) \frac{d \phi_v}{dt} + (m_v + m_g) \frac{d \phi_v}{dt} - (T_1 + T_2) \frac{d \phi_v}{dt} = 0
\]

Using the formulas defining the derivative of a composite function:

\[
\frac{d^2 \phi_j}{dt^2} = \frac{d^2 \phi_j}{dt^2} \frac{d \phi_j}{dt} \frac{d \phi_j}{dt} + \left( \frac{d \phi_j}{dt} \right)^2 \frac{d^2 \phi_j}{dt^2} + \frac{d^2 \phi_j}{dt^2} \frac{d \phi_j}{dt} \frac{d \phi_j}{dt} + \left( \frac{d \phi_j}{dt} \right)^2 \frac{d^2 \phi_j}{dt^2}
\]

and substituting to expression (3) we obtain a second order differential equation:
Figure 1. Distribution and directions of moments in the gripper’s movable part system described in [6] in Figure 1: $I_1c$, $I_2c$, $I_3c$, $I_4c$ - moments of inertia of guide rollers of the belt, $\varphi_{1d}$ - angles of rotation of guide rollers of the belt, $r_1c$, $r_2c$, $r_3c$, $r_4c$, $R_c$ - radii of rolls of the gripper, $M_m$ - drive torque of the motor of rolls, $T_1$ and $T_2$ - boundary frictional forces between the belt and separating layers of the fabric, $m_f$ - mass of the fabric lying on the table (not picked up yet), $m_v$ - mass of the fabric between the foot and belt.

Figure 2. Step of pressing the textile package to the table (A) and a compression of separated layer between the foot and moving part of the gripper (C) described in [6], $m_{sh}$, $m_{sv}$ - weight of the respective elements of the gripper.

Between the angles of rotation $\varphi_{1c}$,...,$\varphi_{4c}$ and the radii of circles $r_1c$, $r_2c$, $r_3c$, $r_4c$, $R_c$ can be formulated thus:

\[
\begin{align*}
\varphi_2R_c &= \varphi_1R_c, \\
\varphi_2r_2c &= \varphi_3r_3c, \\
\varphi_2r_4c &= \varphi_4r_4c.
\end{align*}
\]

(6)

After substituting Equations 6 and 7 into Equation 5, we then obtain a system of Equation 8 of the first order.

For the purpose of numerical calculation of a simplified system of Equations 8, assuming that the guide roller belts are identical, and their moments of inertia $I_{1c}$, $I_{2c}$, $I_{3c}$, $I_{4c}$, $r_{1c}$, $r_{2c}$, $r_{3c}$, $r_{4c}$ and angles of rotation $\varphi_{1c}$, $\varphi_{2c}$, $\varphi_{4c}$ are created equal.

Using relationships (6) and (7) between the angles of rotation and radii of wheels, we can write that:

\[
\begin{align*}
\frac{d\varphi_2}{d\varphi_1} &= \frac{r_{1c}}{R_c}, \\
\frac{d\varphi_3}{d\varphi_2} &= \frac{r_{2c}}{R_c}, \\
\frac{d\varphi_4}{d\varphi_3} &= \frac{r_{4c}}{R_c}.
\end{align*}
\]

(9)

After substituting Equation 9 into the system of Equations 8, the following is obtained:

\[
\frac{d\varphi_3}{d\varphi_2} = \frac{M_m - m_{sh}(N_c + N_h + m_{sh}g)\frac{r_{1c}}{R_c}}{I_{1c} + 3I_{3c}\frac{r_{1c}}{R_c}} + (m_v + m_f)\frac{r_{1c}}{R_c}.
\]

(10)

While manipulating a package of fabrics, the gripper presses the fabric lying on the table in a stack (Figure 2.4) and a layer is separated from the stack between the foot and gripper’s movable part (Figure 2.C).

The motion of the gripper along the vertical direction is initiated by an electric motor 9 with a driving torque $M_m$, and an angle of rotation of the main shaft $\varphi_1$ by means of a screw 7 with a pitch $h_{\varphi_1}$, which transforms the rotary motion of the motor into a vertical translatory one [6].

Analogously the motion of the gripper along the horizontal direction is performed by means of an electric motor 18 with a moment $M_{sh}$ and angle of rotation of the main shaft $\varphi_2$ and a screw 16 of pitch $h_{\varphi_2}$ [6].
Figures 3 and 4 present the distribution of accelerations and forces acting on the mass of the gripper $m_{gh}$ along the horizontal and $m_{gv}$ vertical direction.

The mass $m_{gh}$ consists of gripper elements, i.e., a plate 13 with three belt guide rollers $3$, an electric motor $4$ and a rubber belt $2$ (Figure 2.C and 1 [6]), while the mass $m_{gv}$ consists of a mass $m_{gh}$, a spring $15$ and guides $19$ for stabilization of the gripper motion along the horizontal direction, an electric motor $18$, and casing $14$ (Figure 2.A and 1 [6]).

The dynamic equations of motion for the systems shown in Figure 3 can be presented using the principle of virtual works:

$$M_{m_0} - N_x \frac{dx_v}{d\varphi_h} - m_{h} \frac{d^2x_v}{dt^2} = 0$$

$$M_{m_0} - N_x \frac{dx_v}{d\varphi_h} - m_{h} \frac{d^2x_v}{dt^2} = 0$$

The linear coordinates $x_h$ and $x_v$ and angular coordinates $\varphi_h$ and $\varphi_v$ define the position of the systems with masses $m_{gh}$ and $m_{gv}$. Motion functions $h = x_h(\varphi_h)$ and $v = x_v(\varphi_v)$, which define geometrical relationships between the rotary motion of motors $9$ and $18$ and the horizontal and vertical motion of the gripper, are determined as:

$$x_h = \varphi_h \cdot h_{ih}, \quad x_v = \varphi_v \cdot h_{iv}$$

where $h_{ih}$ and $h_{iv}$ mean the movement of the movable part of the gripper attributable to one rotation of the motor shaft.

From the above equations, the following can be obtained:

$$\frac{dx_v}{dt} = h_{iv}, \quad \frac{d^2x_v}{dt^2} = 0$$

$$\frac{dx_v}{d\varphi_v} = h_{iv}, \quad \frac{d^2x_v}{d\varphi_v} = 0$$

Employing the formula that defines a derivative of the complex function:

$$\frac{d^2x_v}{dt^2} = \frac{d^2\varphi_v}{dt^2} \frac{dx_v}{d\varphi_v} + \frac{d\varphi_v}{dt} \frac{d^2x_v}{dt^2}$$

which after taking into account equations (13) and after substituting to equations (11), we obtain the second order differential equations:

$$m_{h0} \left( \frac{h_{ih}}{2\pi} \right)^2 \frac{d^2\varphi_h}{dt^2} = M_{m_0} - N_x \frac{h_{ih}}{2\pi}$$

$$m_{h0} \left( \frac{h_{iv}}{2\pi} \right)^2 \frac{d^2\varphi_v}{dt^2} = M_{m_0} - N_x \frac{h_{iv}}{2\pi}$$

Equations 15 are then written in the form of first-order equations for computer solution of the method of numerical integration:

$$\frac{d\varphi_h}{dt} = \varphi_h, \quad \frac{d\varphi_v}{dt} = \varphi_v$$

In paper [7] a model of fabric in the form of a linear spring, with three belt guide rollers $3$, an electric motor $4$, and guides $19$ (Figure 2.B) was determined by the formula (5a [6]), for the purposes of this study nonlinear deformation (third degree) of the compression material has been selected. Assumed values of the elasticity constants of the fabric $k_{h1}$, $k_{h2}$, $k_{v1}$ and $k_{v2}$ describing the deformation of the third degree for the sample selected, were determined on the basis of the results given in [8]. The relationships between compressive forces $N_h$ and deformations $u_h$ in the fabric layer between the foot and gripper’s movable part (Figure 4.a) have been assumed in the following form:

$$N_h = \frac{k_{h1}}{g_{nh}} u_h + \frac{k_{h2}}{g_{nh}} u_h^3 + c_{h} du_h$$

where the package of fabrics lying on the table (Figure 4.b) has been assumed in the following form:

$$N_v = \frac{k_{v1}}{g_{nv}} u_v + \frac{k_{v2}}{g_{nv}} u_v^3 + c_{v} du_v$$

for $u_h > 0$ and $L_{ht} > 0.001$

$$N_v = 0$$

for $u_h < 0$

while the package of fabrics lying on the table (Figure 4.b) has been assumed in the following form:

$$N_v = \frac{k_{v1}}{g_{nv}} u_v + \frac{k_{v2}}{g_{nv}} u_v^3 + c_{v} du_v$$

for $u_v > 0$ and $L_{ht} < 0.001$

$$N_v = 0$$

for $u_v < 0$
The increment values of the electromotive forces \( \Delta e_h \) and \( \Delta e_v \) in forces \( \Delta N_h \) and \( \Delta N_v \) relative to the setpoint force \( \Delta N_h = N_h - N_{set} \) and \( \Delta N_v = N_v - N_{set} \), integration step \( \Delta t \) and numbers \( e_h \) and \( e_v \) agreed in order to reach a converging solution:

\[
\begin{align*}
\Delta e_h &= e_h \cdot \Delta N_h \cdot \Delta^* e_v \\
\Delta e_v &= e_v \cdot \Delta N_v \cdot \Delta^* e_h 
\end{align*}
\]

The equations of moments of the motors: driving the rollers which lead belt \( i \), initiate the vertical motion of gripper \( g \), and initiate the horizontal motion of the gripper \( h \), as shown in Figure 1 in [6], are expressed by the formulas:

\[
\frac{dM_m}{dt} = -T_m \left[ c_m \left( \frac{\Omega_m - \omega_0}{2 \pi} \right) - M_m \right] 
\]

where \( T_m \) denotes the motor time constant, \( c_m \) - motor rigidity, \( \Omega_m \) - motor angular velocity at which the moment is equal to zero, \( \omega_0 \) - motor constant, \( \Omega_m \) - motor angular velocity, \( c_m \) - supply voltage, \( K_{m1} \), \( K_{m2} \) - motor constants, and \( \phi_f \) - angle of rotation of the motor.

### Assumptions for the calculation and results

The computer program simulates the work of the gripper, extended by equation (4 - 8), as described in [6] and (1 - 24). Appropriate parameters for the gripper, motors, and textiles have been selected as follows:

- dimensions: \( r_{1c} = 0.0062 \text{ m}, r_{2c} = 0.0138 \text{ m}, r_0 = 0.0116 \text{ m} \), \( x_{0v} = 0.008 \text{ m}, x_{0h} = 0.008 \text{ m} \),
- constant voltage \( r_{1c} = 0.038 \text{ V/\( \text{rad/s} \)} \), constant torque \( K_{t1} = 13.2 \text{ Nm/A} \), resistance \( R = 343 \text{ \Omega} \), inductance \( L = 53 \cdot 10^{-3} \text{ H} \), supply voltage \( e = 25 \text{ V} \) and mass moment of inertia of the rotor \( I_{c1} = 5 \cdot 10^{-8} \text{ kgm}^2 \).

The electric motor initiating the vertical movement of the gripper (item 1, Figure 1 in [6]) is characterised by:

- constant torque \( K_{tv} = 0.038 \text{ V/\( \text{rad/s} \)} \), constant torque \( K_{t2} = 0.33 \text{ Nm/A} \), resistance \( R_v = 343 \text{ \Omega} \), inductance \( L_v = 53 \cdot 10^{-3} \text{ H} \), supply voltage \( e_v = 10 \text{ V} \).

The electric motor initiating the horizontal movement of the gripper (item 18, Figure 1 in [6]) is characterised by:

- constant torque \( K_{th} = 0.038 \text{ V/\( \text{rad/s} \)} \), constant torque \( K_{t1} = 0.33 \text{ Nm/A} \), resistance \( R_h = 343 \text{ \Omega} \), inductance
are obtained.

It was established that

\[ h = 53 \times 10^{-3} \text{ H}, \text{ supply voltage } e_h = 10 \text{ V}. \]

Simulation calculations were carried out for various parameters and the results obtained are shown in the graphs (Figures 5-8).

Graphs (Figure 8) show that compressive forces \( N_h \) & \( N_v \) achieve a maximum value, then decrease and stabilise with time until reaching a constant setpoint value \( N_{set} \).

It has been observed that the stabilisation of parameters up to the value of the force assigned \( N_{set} \) takes place sooner for the vertical direction than for the horizontal one, whereas the values of the compressive forces \( N_h \) and \( N_v \) that cause the deflection of the fabric are close.

These plots illustrate the rheological reaction of the fabric to the gripper operation. Under the influence of the forces the fabric is compressed in the stack \( u_h \), between the gripper and foot \( u_v \) (Figure 7), the values of which, as for forces \( N_h \) & \( N_v \), stabilise in time and cause compression in the range of 0.05%.

Reducing pressures necessary to maintain the fabric is achieved by increasing the contact surface of the respective elements of the gripper.

Entering the edge of the fabric between the respective elements of the gripper was realised by using the endless belt and then pulling the fabric by the force of friction in the direction of the blockage place where the fold entering between the belt and foot is formed.

Computer programs describing the work of grippers for textiles allow analysis of the behavior of the system, which shortens the design process of the device. Modelling studies showed that there is a possibility to choose relevant parameters of these mechanisms.

The model of the gripper developed, which is characterized by an increased working surface, was granted a patent as an invention on 31.01.2008 [9].

### References


### Conclusions

A mathematical model of the gripper separating fabric from a stack has been developed and the phenomena of separation actions of the layer examined. The assumptions made have been confirmed. Thanks to the motors, whose torques are controlled as a function of the size of the pressure, the gripper exerts a sufficient force on the separating fabric to prevent the slipping out of fabric while not exceeding the limit values which may cause damage to the fabric.

A force value \( N_{end} \) was established that the gripper would exert on the separating fabric to prevent it from slipping out while not exceeding the limit force. This is achieved by replacing the clamping spring in appropriate elements of the gripper and having an electric motor whose torque is controlled as a function of the size of the pressure.

The feedback adjusts the value of electromotive forces \( e_h \) and \( e_v \), and consequently the values of the current intensity \( i_h \) and \( i_v \) and driving torques motors \( M_{mh} \) and \( M_{mv} \) are obtained.

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**Figure 7.** Time history of the compression of the material \( u_h \) and \( u_v \) in m.

**Figure 8.** Time history of the compressing force on the fabric layer located between the gripper and foot \( N_h \) in N and of the stack of the fabric \( N_v \) in N.