Multicriterial Optimisation of the Manufacturing Process of a Spindle Working in a Ring Spinning Frame

Abstract
In the paper a method of the best variant selection of the manufacturing process of a spindle with a collapsed balloon crown of a ring spinning frame is presented. The method implements the original optimisation procedure based on the given criteria, taking into consideration the importance of the criteria. The unit manufacturing cost and five criteria of the manufacturing quality were taken as the assessment criteria, and the knowledge of experts was used to determine the importance of the criteria given for assessment. Each expert built his own importance matrix of the assessment criteria, comparable in pairs, using the Saaty method. On the basis of a cumulative matrix the weights of individual criteria were determined. The criteria of the assessment obtained from calculations and measurements were normalised. In the next stage, normalised decisions were created by raising each assessment to a power equal to the corresponding weight. In the last stage of the proceedings a single, optimal alignment comprising the smallest s-th components of the individual decisions d1, d2, ..., dm was created. The variant which corresponds to the largest component of the optimal alignment is assumed as the best.

Key words: multicriteria optimisation, planning manufacturing processes, collapse balloon spindle, a ring spinning frame.

Introduction
Diversity of means and methods of surface treatment and their selection leads to a situation where components, identical or similar in shape, dimensions or accuracy, are often produced according to different manufacturing processes, differing from each other in labour consumption and costs, additionally ensuring different manufacturing quality of the workpiece, and as a consequence, better or worse functional quality. In connection with it a complex multivariate task of planning and the selection of the most rational variant of the manufacturing process of the product arises [1].

Considering the issue of the selection of the most rational variant of the manufacturing process, both the optimisation of the parameters and the structure of the manufacturing processes should be taken into account [2, 3]. The issue of optimisation of the structure of manufacturing processes has achieved an extensive bibliography, which was discussed, among others, in detail in the works [4 - 6]. However, the issue of optimisation of the structure of manufacturing processes with respect to two or more criteria has been discussed in a few works only [7 - 9].

As a starting point for the optimisation, the designation of a set of variants (alternatives) of the manufacturing process of a considered workpiece, assessed in the light of the criteria determined, was assumed. In the most general case, except the criteria having a deterministic (stated precisely, sharp ones) and probabilistic-statistical character, ones with a fuzzy (subjective) character can occur [10]. Optimisation criteria are often treated in conventional models as deterministic – e.g. cost, and often must be considered in the planning phase of the manufacturing process as non-deterministic, and hence, for instance, as subjective point assessments [8, 11] or fuzzy assessments [10]. In general, however, in the majority of cases, in the course of optimisation of manufacturing processes of a similar product, optimisation criteria with a probabilistic-statistical character, in order to simplify the proceeding, are treated as deterministic ones, e.g. surface roughness parameters.

The objective of the present work is the presentation of a multi-criteria optimisation method for the manufacturing process with respect to criteria obtained from calculations and measurements, taking into consideration their importance for the selection of the best variant of the manufacturing process of a spindle with a collapsed balloon crown of a ring spinning frame.

Method of assessment of the variants due to the criteria adopted and their importance

Input data in the method developed are:
- number of variants of the manufacturing process \(- s (s = 1, ..., n)\),
- number of criteria \(- t (t = 1, ..., m)\),
- elements of the importance matrix of individual criteria \(B = [b_{ij}]\),
- elements of \(C = [c_{ij}]\) table, being normalised assessments of the s-th variant according to the t-th criterion.

Let \(A\) denote a permissible set of variants (alternatives) of the manufacturing process:
\[
A = \{a_1, a_2, ..., a_n\}
\]  
(1)

and \(K\) – a set of assessments obtained from calculations or measurements:
\[
K = \{k_1, k_2, ..., k_m\}
\]  
(2)
The importance matrix of individual criteria \( \mathbf{B} \) is then created:

\[
\mathbf{B} = [b_{ij}] \quad i = 1, \ldots, m, \quad j = 1, \ldots, m \quad (3)
\]

Matrix \( \mathbf{B} \) is evaluated with the use of the Saaty method [12], consisting in the comparison of the successive pairs of the criteria. Individual values \( b_{ij} \) of the matrix were taken in the following way:

- \( b_{ij} = 1 \) – when \( k_i \) and \( k_j \) are equally important,
- \( b_{ij} = 3 \) – when \( k_i \) is a little bit more important than \( k_j \),
- \( b_{ij} = 5 \) – when \( k_i \) is more important than \( k_j \),
- \( b_{ij} = 7 \) – when \( k_i \) is distinctly more important than \( k_j \),
- \( b_{ij} = 9 \) – when \( k_i \) is absolutely more important than \( k_j \),
- \( b_{ij} = 2, 4, 6, 8 \) – intermediate values between the above situations.

Moreover it was assumed that \( b_{ij} = 1/b_{ji} \), and for the \( i = j \) value of \( b_{ij} = 1 \).

In the case of a few experts, the construction of the importance matrix of criteria \( \mathbf{B} \) is performed in the following way:

- each of the experts creates matrix \( \mathbf{B} \) individually,
- from the matrices obtained, called partial matrices, a single collective importance matrix of the criteria is created (any item of the matrix above the main diagonal is calculated as an arithmetic mean from the appropriate items of the partial matrices, while the items under the main diagonal become the converses of corresponding items located above the main diagonal).

Since the importance matrix of the criteria is created as the result of the comparison of successive pairs of the criteria, it follows that this matrix is a square matrix of a size equal to the number of criteria. The matrix should fulfill, at least approximately, the condition of consistence [12]:

\[
CI = \frac{\lambda_{\text{max}} - m}{m-1} \leq 0.1 \quad (4)
\]

where: \( \lambda_{\text{max}} \) – scalar denoting the maximal eigenvalue of matrix \( \mathbf{B} \), \( m \) – number of criteria, and also the rank of matrix \( \mathbf{B} \).

From the Saaty method it is seen that satisfactory fulfillment of the condition \( CI \leq 0.1 \) assures satisfactory adequacy of the method, in which the eigenvalues and eigenvectors of matrix \( \mathbf{B} \) are present.

The next step of the method developed consists in the evaluation of the table \( \mathbf{C} = [c_{ij}] \), performed on the basis of calculations or measurements of the values of the criteria given for the assessment of individual variants of the manufacturing process (Table 1).

Assessments \( c_{st} \), obtained from calculations or measurements, are subjected to normalisation, making use of the following dependency:

\[
c_{st} = 0.1 + \left( \frac{\max_{s,t} (c_{st}) - \min_{s,t} (c_{st})}{\max_{s,t} (c_{st}) - \min_{s,t} (c_{st})} \right) \cdot 1.25 \quad (5)
\]

where: \( c_{st} \) – assessments of the variants analysed against individual criteria, \( s = 1, \ldots, n; \quad t = 1, \ldots, m; \quad m \) – number of variants; \( m \) – number of criteria.

The normalized assessments \( c_{st}^* \), obtained according to formula (5), are fractions from interval \(<0.1; 0.9>\). Such a method of normalisation excludes extreme assessments, which are very close to 0 and to 1.

Afterwards normalised assessments \( c_{st}^* \) are converted depending on the method of optimisation, i.e. depending on the situation where a given criterion should undergo minimisation or maximisation, according to the following formula:

\[
c_{st}^* = (1 - k_s) \cdot (1 - c_{st}^*) + k_s \cdot c_{st}^* \quad (6)
\]

where: \( k_s \) for \( t = 1, \ldots, m \) is a scalar with values 0 or 1.

If \( k_s = 1 \) the best variant is that with the highest value of the assessment according to the \( t \)-th criterion, \( k_s = 0 \) – the best variant is that with the lowest value of the assessment according to the \( t \)-th criterion.

On the basis of the normalised and transformed values evaluated, a table of assessments was created for individual criteria and for each variant of the planned manufacturing process analysed (Table 2).

The next step of the correct phase of searching, after the best (optimal) variant, is the evaluation of eigenvector \( \mathbf{Y} \), which fulfills the following matrix equation:

\[
\mathbf{B} \cdot \mathbf{Y} = \lambda_{\text{max}} \cdot \mathbf{Y} \quad (7)
\]

where: \( \mathbf{B} \) – cumulative importance matrix of criteria, \( \mathbf{Y} \) – eigenvector, in the above equation, a column matrix, \( \lambda_{\text{max}} \) – scalar denoting maximal eigenvalue of matrix \( \mathbf{B} \).

Therefore a vector for which equation \( \mathbf{B} \cdot \mathbf{Y} = \lambda_{\text{max}} \cdot \mathbf{Y} \) is fulfilled, for possibly the maximal eigenvalue \( \lambda = \lambda_{\text{max}} \) was searched for. The sought-after vector features as many coordinates as criteria present.

The coordinates should comply with an additional condition where the sum of the coordinates should be equal to the number of criteria [11].

\[
\sum_{i=1}^{n} y_i = m \quad (8)
\]

where: \( y_t \) – \( t \)-th coordinate of eigenvector \( \mathbf{Y} \).

The coordinates of the eigenvector are also the weights of individual criteria and are marked with characters \( w_1, w_2, \ldots, w_m \). Each of the weights expresses the importance of the criterion corresponding to this weight, whereas the higher the value of the \( t \)-th weight, the higher the importance of the \( t \)-th criterion.

The next step of the method developed is based on the Yager method [11] and consists in the creation of normalised decisions through raising each component of normalized assessments to a power equal to the corresponding weight. In a general form it can be written in the following way:

\[
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\]
The last step of the method developed consists in the creation of an optimal alignment of the variants with respect to the criteria given for assessment, on the basis of which the optimal variant of the process is selected, i.e. a variant which best fulfills all of the criteria for assessment. Optimal alignment within the method developed, similar to the Yager method [11], is a decision of the minimum type. The \( r \)-th component of the optimal alignment (i.e. a component corresponding to the \( r \)-th variant of the manufacturing process) is the smallest \( r \)-th component of all individual decisions \( d_1, d_2, ..., d_m \).

By marking the optimal alignment and its components with capital letters, \( D \), it is possible to write the dependency presented in Table 4.

\[
d_j = \sum_{i=1}^{n} (c_i^{jm}) y_i \tag{9}
\]

After transcription, formula (9) takes the form presented in Table 3.

The spinning spindle accomplishes a twist of yarn and forms yarn packing on a ring spinning frame. The ring spinning frames are destined for the production of yarn with a thickness of 50 tex mainly used a spindle topped with a spindle crown in its upper part, positioned at a distance of 6 – 12 mm from the eylet of the yarn guide. A complete spinning spindle (Figure 1) consists of three main sub-assemblies: a spindle coating with a needle and top (1), a brake (2) and spindle bearing unit (3).

One of the main components of the spindle is its coating, which is produced from the EN-AW 2024 (AlCu4Mg1) alloy; its neck should be machined with an IT10 tolerance to a surface roughness of \( R_a \leq 0.40 \mu m \).

**Table 3. Values of the assessments of each variant after consideration of the importance of the criteria.**

<table>
<thead>
<tr>
<th>Variants</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>...</th>
<th>( a_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 )</td>
<td>( c_{11}^{a_1} w_1 )</td>
<td>( c_{12}^{a_1} w_1 )</td>
<td>( c_{13}^{a_1} w_1 )</td>
<td>...</td>
<td>( c_{1n}^{a_1} w_1 )</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>( c_{21}^{a_2} w_2 )</td>
<td>( c_{22}^{a_2} w_2 )</td>
<td>( c_{23}^{a_2} w_2 )</td>
<td>...</td>
<td>( c_{2n}^{a_2} w_2 )</td>
</tr>
<tr>
<td>( k_3 )</td>
<td>( c_{31}^{a_3} w_3 )</td>
<td>( c_{32}^{a_3} w_3 )</td>
<td>( c_{33}^{a_3} w_3 )</td>
<td>...</td>
<td>( c_{3n}^{a_3} w_3 )</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>( k_m )</td>
<td>( c_{m1}^{a_m} w_m )</td>
<td>( c_{m2}^{a_m} w_m )</td>
<td>( c_{m3}^{a_m} w_m )</td>
<td>...</td>
<td>( c_{mn}^{a_m} w_m )</td>
</tr>
</tbody>
</table>

**Table 4. Optimal alignment of the variants in view of the criteria given for assessment**

<table>
<thead>
<tr>
<th>Variants</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>...</th>
<th>( a_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_1 )</td>
<td>( D_1 )</td>
<td>( D_2 )</td>
<td>( D_3 )</td>
<td>...</td>
<td>( D_n )</td>
</tr>
</tbody>
</table>

During spinning operations, the yarn, moving at a velocity of up to 35 m/min, as a result of local friction generated by tension of a value within the range of 0.10 – 0.30 N, and mainly due to its stoppage by one of the teeth of the crown, carves deep and considerably wide spiral grooves on the neck of the coating [13, 14], which are the cause of the increased number of end breaks of the yarn.

Observations performed within an industrial environment have confirmed that the neck of the coating is the least durable component of the spindle coating. Its operational lifetime was included within the interval of about 9 000 to about 14 000 working hours during the production of yarn composed from raw material with blend ratios of 30/70 and 55/45 for polyester/wool fibres, with the time of the failure-free operation of the bearing unit equal to 36 000 working hours, as warranted by the manufacturer.

To improve this situation, i.e. to improve the operational lifetime of the spindle coating at the predetermined manufacturing costs, and with functional parameters of the spindle maintained, eleven variants of the manufacturing process of the spindle with a collapsed balloon crown were developed and analysed.

**Set of variants**

In general, all of the variants are characterised by the fact that the contour turning operation of the coating, marked as I (the first), is accomplished before the connection of the coating with the needle and top, whereas the contour turnings marked as II (the second), III (the third) and IV (the fourth) are accomplished after the connection of the coating with the needle and top, taking the chamfer (external center hole) on the pin of the neck of the coating and the chamfer on the top as reference. Turning operations of the pin fixing the position of the crown are performed simultaneously with the remaining part of the coating turning [8].

**Example of multicriteria optimisation of the manufacturing processes of a spindle with a collapsed balloon crown of a ring spinning frame**

The spinning spindle accomplishes a twist of yarn and forms yarn packing on a ring spinning frame. The ring spinning frames destined for the production of yarn with a thickness of 50 tex mainly use a spindle topped with a spindle crown in its upper part, positioned at a distance of 6 – 12 mm from the eylet of the yarn guide. A complete spinning spindle (Figure 1) consists of three main sub-assemblies: a spindle coating with a needle and top (1), a brake (2) and spindle bearing unit (3).
Eleven variants of the manufacturing process of the spindle with a collapsed balloon crown were analysed, differing from each other in the surface treatment and finishing treatment of the neck of the spindle’s coating. These variants are presented in the form of a graph-tree (Figure 2) and are depicted in Table 5.

1. Diamond turning without cooling, with the following parameters: \(v_c = 3.23 \text{ m/s}, f = 0.03 \text{ mm/rotation}, \ a_p = 0.15 \text{ mm}, \) turning cutter, type 5529 with a diamond cutting edge of geometry: \(\alpha = 2^\circ 30', \gamma = 6^\circ, \ \kappa_f = 4^\circ, \) \(\kappa_2 = 35^\circ\) and a fillet radius of the main cutting edge \(r_s = 1.2 \text{ mm}\).

2. Diamond turning (with the same parameters as variant 1) and anodic oxidation.

3. Diamond turning (with the same parameters as variant 1), anodic oxidation and grinding with PS 20 corundum abrasive paper, grain size 600, with the following parameters: \(v_c = 3.11 \text{ m/s} \) and unit pressure \(p_n \approx 0.15 \cdot 10^5 \text{ Pa}\).

4. Grinding with HTJ-13-3 corundum abrasive cloth, grain size 150, and next with grain size 320, with the following parameters: \(v_c = 3.11 \text{ m/s} \) and unit pressure \(p_n \approx 0.45 \cdot 10^5 \text{ Pa}\).

5. Grinding with HTJ-13-3 corundum abrasive cloth, grain size 150, and next with grain size 320 (parameters the same as in variant 4) and anodic oxidation.

6. Grinding with HTJ-13-3 corundum abrasive cloth, grain size 150, and next with grain size 320 (parameters the same as in variant 4), anodic oxidation and grinding with PS 20 corundum abrasive paper, grain size 600, with the following parameters: \(v_c = 3.11 \text{ m/s} \) and unit pressure \(p_n \approx 0.45 \cdot 10^5 \text{ Pa}\).

7. Burnishing with a disc of diameter \(D_k = 40 \text{ mm}, \) and radius in the axial cross-section \(r_k = 9 \text{ mm}, \) with a pressure load of the disc \(F = 0.30 \text{ kN}, \) with feedrate \(f = 0.10 \text{ mm/rotation}, \) at a circumferential speed of the coating \(v_c = 2.07 \text{ m/s}, \) number of passages of the disc \(i = 1, \) and cooling and lubricating with machine oil 10.

8. Burnishing (with parameters as in variant 7) and anodic oxidation.

9. Burnishing (with parameters as in variant 7), anodic oxidation and grinding with PS 20 corundum abrasive paper of grain size 600, with the following parameters: \(v_c = 3.11 \text{ m/s} \) and unit pressure \(p_n \approx 0.45 \cdot 10^5 \text{ Pa}\).

10. Plasma spraying of the layer of \(\text{Al}_2\text{O}_3\) ceramic powder and thickness 0.10 – 0.15 mm (and hardness of about 14,400 MPa and grain size 20 – 90 µm).

11. Plasma spraying of the layer of \(\text{Al}_2\text{O}_3\) ceramic powder and thickness 0.10 – 0.15 mm (and hardness of about 14,400 MPa and grain size 20 – 90 µm), grinding with HTJ-13-3 corundum abrasive cloth, grain size 320 and PS 20 corundum abrasive paper, grain size 600, with the following parameters: \(v_c = 3.11 \text{ m/s} \) and unit pressure \(p_n \approx 2.0 \cdot 10^5 \text{ Pa}\).

Hard anodic oxidation [8] was performed in a solution of electrolyte with the following composition (by weight): sulfurous acid – 6%, sulfosalicilic acid – 3%, lactic acid – 2%, glycerol – 2%, aluminum sulfate – 0.1%, and distilled water as the remainder. Electrolyte bath mixing was performed with compressed air. Conditions of the anodic oxidation were as follows: DC current voltage
Table 5. Description of the manufacturing process of a spindle with a collapsed balloon crown of a ring spinning frame.

<table>
<thead>
<tr>
<th>No. of operation</th>
<th>Name of the operation</th>
<th>Machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Cutting the ø 36 bar to length 453 mm</td>
<td>Turning lathe RD28/40x700</td>
</tr>
<tr>
<td>20</td>
<td>Turning and milling the pin</td>
<td>Turning lathe with SINUMERIK 810D n.c. control</td>
</tr>
<tr>
<td>30</td>
<td>Turning the cone, facing, drilling and broaching</td>
<td>Turning lathe with SINUMERIK 810D n.c. control</td>
</tr>
<tr>
<td>40</td>
<td>Grinding chamfer 90° (datum surface) on the pin.</td>
<td>Grinder NWK NLW 30054</td>
</tr>
<tr>
<td>50</td>
<td>Turning (I) the coating with excess material for 4 passages</td>
<td>Turning lathe with SINUMERIK 810D n.c. control</td>
</tr>
<tr>
<td>60</td>
<td>Pushing the needle to the coating</td>
<td>Hand press</td>
</tr>
<tr>
<td>70</td>
<td>Grinding the cone under the ø 25.07 mm top</td>
<td>Grinder NWK NLW 30054</td>
</tr>
<tr>
<td>80</td>
<td>Pushing the top on the coating</td>
<td>Hand press</td>
</tr>
<tr>
<td>90</td>
<td>Grinding chamfer 90° (datum surface) on the pin</td>
<td>Grinder NWK NLW 30054</td>
</tr>
<tr>
<td>100</td>
<td>Turning (II) the coating with 1 mm allowance</td>
<td>Turning lathe with SINUMERIK 810D n.c. control</td>
</tr>
<tr>
<td>110</td>
<td>Grinding chamfer 90° (datum surface) on the pin.</td>
<td>Grinder NWK NLW 30054</td>
</tr>
<tr>
<td>120</td>
<td>Turning (III) the coating with 0.5 mm allowance</td>
<td>Turning lathe with SINUMERIK 810D n.c. control</td>
</tr>
<tr>
<td>130</td>
<td>Grinding chamfer 90° (datum surface) on the top.</td>
<td>Grinder NWK NLW 30054</td>
</tr>
<tr>
<td>140</td>
<td>Grinding chamfer 90° (datum surface) on the top</td>
<td>Grinder NWK NLW 30054</td>
</tr>
<tr>
<td>150</td>
<td>Finishing turning (IV) the coating</td>
<td>Turning lathe with SINUMERIK 810D n.c. control</td>
</tr>
<tr>
<td>160</td>
<td>Facing the pin and drilling for thread M6</td>
<td>Turning lathe TUB 32x1000</td>
</tr>
<tr>
<td>170</td>
<td>Tapping the hole M6xl4</td>
<td>Tapping machine GWD 27</td>
</tr>
<tr>
<td>180</td>
<td>Polishing the top and coating</td>
<td>Turning lathe TUB 32x1000</td>
</tr>
<tr>
<td>190</td>
<td>Milling 6 holes R = 4.56 mm</td>
<td>Special milling machine H 36 „Susen“</td>
</tr>
<tr>
<td>200</td>
<td>Blunting sharp edges and assembly of caps</td>
<td>Assembly station</td>
</tr>
<tr>
<td>210</td>
<td>Finishing turning (IV) the coating and neck thereof before diamond turning</td>
<td>Turning lathe with SINUMERIK 810D n.c. control</td>
</tr>
<tr>
<td>220</td>
<td>Finishing turning (V) the neck of the coating with diamond cutting edge</td>
<td>Turning lathe with SINUMERIK 810D n.c. control</td>
</tr>
<tr>
<td>230</td>
<td>Finish turning (IV) the coating, turning the neck of the coating for Al₂O₃ plasma spraying</td>
<td>Turning lathe with SINUMERIK 810D n.c. control</td>
</tr>
<tr>
<td>240</td>
<td>Anodic oxidation of the neck of the coating for length of about 100 mm</td>
<td>Station to anodic oxidation</td>
</tr>
<tr>
<td>250</td>
<td>Grinding the neck of the coating with abrasive cloth P320</td>
<td>Turning lathe TUB 32x1000</td>
</tr>
<tr>
<td>260</td>
<td>Grinding the neck of the coating with abrasive paper P600</td>
<td>Turning lathe TUB 32x1000</td>
</tr>
<tr>
<td>270</td>
<td>Burning the neck for length 86 mm</td>
<td>Turning lathe TUB 32x1000</td>
</tr>
<tr>
<td>280</td>
<td>Spraying the neck of the coating with alumina, Al₂O₃</td>
<td>Plastic spraying station</td>
</tr>
<tr>
<td>290</td>
<td>Grinding with abrasive cloth P150 and P320</td>
<td>Turning lathe TUB 32x1000</td>
</tr>
<tr>
<td>300</td>
<td>Straightening the top by punching operation</td>
<td>Straightening machine</td>
</tr>
<tr>
<td>310</td>
<td>Grinding the neck where whipping at the top is above the 0.08 mm</td>
<td>Grinder NWK NLW 30054</td>
</tr>
<tr>
<td>320</td>
<td>Assembly of the crown</td>
<td>Assembly station</td>
</tr>
</tbody>
</table>

25 - 60 V, current density 3 A/dm², temperature of electrolyte solution - about 6°C, and time of anodic oxidation - about 60 min. Before the hard anodic oxidation the following operations of surface preparation were performed: degreasing in organic solvent and etching in 5% solution of sodium hydrate for 2 minutes.

Plasma spraying was performed with the use of a plasma device - PLANCER PN-110 (National Centre for Research Nuclear, Swierk, Poland) type with the following parameters: power of the burner 30 kW, flow rate of argon and 5 - 10% of hydrogen – 2.5 m³/h, flow rate of argon transporting the powder – 0.30 m³/h, and distance between the burner and work piece – 25 mm.

Set of criteria for assessment

To assess the variants of the manufacturing process of a spindle with a collapsed balloon crown, the following six deterministic criteria were given for assessment:

- unit manufacturing cost $K_w$, EUR,
- mean square deviation of profile roughness $R_q$, μm,
- mean square gradient of profile roughness $R_{qv}$, rad,
- average curvature radius of profile peaks (irregularities) $r_w$, μm,
- maximal hardness on the face of the surface layer $H_V$, MPa,
- hardening depth of the surface layer $g^{(w)}$, μm.

Criteria related to the quality of the yarn (cv of the yarn, number of end breaks per unit length, etc.) were not taken into consideration in the course of definition of this optimisation task because investigations performed within industrial conditions did not allow unequivocal confirmation of the effect of the geometrical structure of the coating on the above-mentioned criteria.

To assess the geometrical structure of the surface (SGP in short), three parameters: $R_q$, $R_{qv}$ and $r_w$ were taken into consideration, because values of the linear correlation coefficient $r$ calculated between these parameters and the coefficient of kinetic friction $μ_k$ of the yarn against the neck of the spindle with a collapsed balloon crown of a ring spinning frame were the highest [8]. Recordings and measurements of geometrical structure parameters of the surface were performed with the use of a Talysurf 6 (Rank Taylor Hobson, GB) profile gauge with a measuring probe of conical shape and imaging nose radius of $r_{nm} = 2$ μm, wherein the average corner radius of the surface’s irregularities peaks $r_w$ was evaluated on the basis of a parabolic approximation of the profiles’ peaks. Among all of the local peaks of the filtered profile, nine maximal profiles with a shape typical for a given profile were taken for assessment, making use of the possibility of graphic assessment of the approximate quality of selected fragments of the profile. The approximation consisted in the selection of nine points: one point lying at the peak and four points, both from the RH and LH side of the peak. The distance between the points was equal to the stage of digitalisation, i.e. 0.5 μm. The fillet radius of the peak of the parabola corresponded to the fillet radius of the peak of the irregularity of the profile. Measurements of the above surface roughness parameters were performed for at least five spindles for each type (variant) of surface and finishing treatment, and measurements were
Table 6. Values of assessment criteria after normalisation for individual variants of the manufacturing process.

<table>
<thead>
<tr>
<th>Variants</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$a_7$</th>
<th>$a_8$</th>
<th>$a_9$</th>
<th>$a_{10}$</th>
<th>$a_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>0.8771</td>
<td>0.6592</td>
<td>0.6542</td>
<td>0.8985</td>
<td>0.6805</td>
<td>0.6756</td>
<td>0.9000</td>
<td>0.6820</td>
<td>0.6789</td>
<td>0.1124</td>
</tr>
<tr>
<td>$k_2$</td>
<td>0.8389</td>
<td>0.4279</td>
<td>0.6678</td>
<td>0.8642</td>
<td>0.5961</td>
<td>0.7573</td>
<td>0.9000</td>
<td>0.6774</td>
<td>0.7667</td>
<td>0.1000</td>
</tr>
<tr>
<td>$k_3$</td>
<td>0.7392</td>
<td>0.4154</td>
<td>0.5366</td>
<td>0.7809</td>
<td>0.4133</td>
<td>0.5688</td>
<td>0.9000</td>
<td>0.4071</td>
<td>0.6076</td>
<td>0.1000</td>
</tr>
<tr>
<td>$k_4$</td>
<td>0.2003</td>
<td>0.1133</td>
<td>0.1396</td>
<td>0.2759</td>
<td>0.1206</td>
<td>0.1565</td>
<td>0.9000</td>
<td>0.1275</td>
<td>0.1721</td>
<td>0.1000</td>
</tr>
<tr>
<td>$k_5$</td>
<td>0.1000</td>
<td>0.2490</td>
<td>0.2551</td>
<td>0.1030</td>
<td>0.2460</td>
<td>0.2506</td>
<td>0.1183</td>
<td>0.2338</td>
<td>0.2360</td>
<td>0.0000</td>
</tr>
<tr>
<td>$k_6$</td>
<td>0.1000</td>
<td>0.2920</td>
<td>0.2600</td>
<td>0.1320</td>
<td>0.2780</td>
<td>0.2538</td>
<td>0.2120</td>
<td>0.2440</td>
<td>0.2280</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

In the second stage of normalisation, it was considered whether a given criterion in the optimisation task should be maximised or minimised. To do it, the function presented by formula (6) is used.

In the example analysed, the manufacturing cost of one piece $K_n$, the mean square deviation of the profile roughness $R_q$ and the mean square gradient of the profile roughness $R_q/l$ belong to the minimised criteria (for which $k_{er} = 0$), while the fillet radius $r_n$, maximal hardness of the surface layer $HV$ and the depth of the strengthening (hardening) of the surface layer $g^{(ww)}$ belong to maximised criteria ($k_{er} = 1$).

Values of the assessments after normalisation and transformation, depending on the method of optimization, for individual criteria and each variant of the manufacturing process of the spindle are presented in Table 6.

Three experts were engaged to evaluate the importance of individual criteria used for assessment of the set of variants of the manufacturing process of a spindle with a collapsed balloon crown analysed, where the 1st expert was a specialist-design engineer of textile machinery, the 2nd expert—a specialist from the area of planning manufacturing processes, and the 3rd expert was a specialist from the area of manufacturing costs and economic analyses. To assess the importance of the given criteria, each expert built his own importance matrix of the assessment, comparable pairwise, using the Saaty method [12] (Tables 7, 8 and 9).

On the basis of the matrices constructed, called partial matrices, a cumulative matrix (Table 10) was created with the items located over the main diagonal being arithmetic means of the appropriate items of individual partial matrices. On the other hand, items of the matrix under the main diagonal are inverses of the values corresponding to the items over the main diagonal. The cumulative matrix is the basis to evaluate the importance (weights) of individual criteria given for the assessment of the variants of the manufacturing process of the spindle analysed.

In the next step, eigenvalues of the cumulative importance matrix of criteria B were calculated with the use of the Power method [16], comparing its determinant to zero and solving the equation of $n = 6$.
The solution of Equation 12 are eigenvalues \( \lambda \) of matrix B:

\[
\begin{align*}
\lambda_1 & = 6.2373; \quad 0.0203 + 1.1808i; \\
\lambda_2 & = 0.0203 - 1.1808i; -0.1895; -0.0442 + 0.2794i; -0.0442 - 0.2794i.
\end{align*}
\]

Hence the maximal eigenvalue of matrix B sought amounts to: \( \lambda_{\text{max}} = 6.2373 \)

Verification of the condition of the consistence of matrix B:

\[
CI = \frac{\lambda_{\text{max}} - m}{m - 1} = \frac{6.2373 - 6}{6 - 1} = 0.04746 \leq 0.1
\]

And hence, the condition of consistence, approximately, is fulfilled because \( CI = 0.04746 \leq 0.1 \). Next for the maximal eigenvalue \( \lambda_{\text{max}} = 6.2373 \) of matrix B and the condition that sums the coordinates of eigenvector \( Y \), which should be equal to the number of criteria (formula 8), values of these coordinates \( y_k \) \((k = 1, \ldots, m)\), solving the following system of equations were evaluated using the method of Gauss elimination (see Equation 14).

The solution of the system of Equation 14 are values:

\[
\begin{align*}
y_1 &= 1.4182; y_2 = 1.6080; \\
y_3 &= 0.3500; y_4 = 0.8565; \\
y_5 &= 1.6099; y_6 = 0.1574,
\end{align*}
\]

satisfying the equation:

\[
1.4182 + 1.6080 + 0.3500 + 0.8565 + 1.6099 + 0.1574 = 6
\]

Coordinates \( y_k \) are simultaneously the weights \( w_t \) of individual criteria.

The next stage of the method consists in the creation of normalised decisions through raising each component of successive assessments to a power equal to the corresponding weight, according to formula (9). Values of the normalised decisions for each of the variants, taking into consideration individual criteria, are presented in Table 11.

The last stage of the method developed consists in the creation of a single optimal alignment, on the basis of such the best variant of the manufacturing process of the spindle is selected, i.e. such a process which best complies with all of the criteria given for the assessment. The optimal alignment in this method is the decision minimum. The \( s \)-th component of the optimal alignment, i.e. the component corresponding to the \( s \)-th variant of the manufacturing process, is the smallest \( s \)-th component of individual decisions \( d_1, d_2, \ldots, d_m \) (formula 10). The value of the optimal alignment for individual variants is presented in Table 12.

The variant which corresponds to the biggest component of the optimal alignment is considered the best (optimal variant):

\[
a_{(s_\text{opt})} = \max_s D_s = 0.110899 \quad (a_s) \quad (16)
\]

In this case, variant \( a_7 \) should be taken as the best variant, because it corresponds to the maximal value of the optimal alignment, equal to 0.110899. In this variant, the neck of the coating of the spindle is subjected to the operations of finish turning with diamond cutting edge, and next hard anodic oxidation, followed by grinding with PS 20 corundum abrasive paper, grain size 600.

### Conclusions

The method of proceeding optimisation presented \( i \) consists in replacing, by the well-known Yager method, criteria in the form of point assessments defined by experts with the values of the criteria obtained from calculations and measurements. To eliminate units of measure-

### Table 10. Cumulative importance matrix of criteria.

<table>
<thead>
<tr>
<th>( k_j )</th>
<th>( k_1 )</th>
<th>( k_2 )</th>
<th>( k_3 )</th>
<th>( k_4 )</th>
<th>( k_5 )</th>
<th>( k_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 )</td>
<td>1</td>
<td>1.0972</td>
<td>3.4000</td>
<td>2.1481</td>
<td>1.0925</td>
<td>5.5000</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>0.91139</td>
<td>1.0000</td>
<td>5.6667</td>
<td>2.8333</td>
<td>0.8333</td>
<td>8.6667</td>
</tr>
<tr>
<td>( k_3 )</td>
<td>0.29412</td>
<td>0.17647</td>
<td>1.0000</td>
<td>0.2611</td>
<td>0.16984</td>
<td>4.0000</td>
</tr>
<tr>
<td>( k_4 )</td>
<td>0.18182</td>
<td>0.11539</td>
<td>0.2500</td>
<td>0.14286</td>
<td>0.11111</td>
<td>10.0000</td>
</tr>
</tbody>
</table>

### Table 11. Values of the assessment of individual variants after consideration of the importance of the criteria.

<table>
<thead>
<tr>
<th>Variants</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
<th>( a_5 )</th>
<th>( a_6 )</th>
<th>( a_7 )</th>
<th>( a_8 )</th>
<th>( a_9 )</th>
<th>( a_{10} )</th>
<th>( a_{11} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 )</td>
<td>0.8304</td>
<td>0.5538</td>
<td>0.5478</td>
<td>0.8592</td>
<td>0.5794</td>
<td>0.5735</td>
<td>0.8612</td>
<td>0.5811</td>
<td>0.5751</td>
<td>0.0451</td>
<td>0.0382</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>0.7539</td>
<td>0.2554</td>
<td>0.5225</td>
<td>0.7907</td>
<td>0.4533</td>
<td>0.6396</td>
<td>0.8442</td>
<td>0.5346</td>
<td>0.8400</td>
<td>0.0247</td>
<td>0.2401</td>
</tr>
<tr>
<td>( k_3 )</td>
<td>0.8996</td>
<td>0.7353</td>
<td>0.8042</td>
<td>0.9171</td>
<td>0.7340</td>
<td>0.8308</td>
<td>0.9638</td>
<td>0.7301</td>
<td>0.8245</td>
<td>0.4667</td>
<td>0.6613</td>
</tr>
<tr>
<td>( k_4 )</td>
<td>0.2523</td>
<td>0.1548</td>
<td>0.1852</td>
<td>0.3319</td>
<td>0.1633</td>
<td>0.2032</td>
<td>0.9137</td>
<td>0.1714</td>
<td>0.2216</td>
<td>0.1391</td>
<td>0.1941</td>
</tr>
<tr>
<td>( k_5 )</td>
<td>0.0224</td>
<td>0.1067</td>
<td>0.1109</td>
<td>0.0258</td>
<td>0.1044</td>
<td>0.1077</td>
<td>0.0322</td>
<td>0.0964</td>
<td>0.0978</td>
<td>0.8440</td>
<td>0.8440</td>
</tr>
<tr>
<td>( k_6 )</td>
<td>0.6960</td>
<td>0.8239</td>
<td>0.8009</td>
<td>0.7271</td>
<td>0.8166</td>
<td>0.8058</td>
<td>0.7834</td>
<td>0.8009</td>
<td>0.7924</td>
<td>0.9836</td>
<td>0.9780</td>
</tr>
</tbody>
</table>

### Table 12. Optimal alignment of variants in view of the criteria given for assessment.

<table>
<thead>
<tr>
<th>Variants</th>
<th>( D_s )</th>
<th>( a_{10} )</th>
<th>( a_{11} )</th>
<th>( a_{10} )</th>
<th>( a_{11} )</th>
<th>( a_{10} )</th>
<th>( a_{11} )</th>
<th>( a_{10} )</th>
<th>( a_{11} )</th>
<th>( a_{10} )</th>
<th>( a_{11} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_s )</td>
<td>0.0246</td>
<td>0.1067</td>
<td>0.1109</td>
<td>0.0258</td>
<td>0.1044</td>
<td>0.1077</td>
<td>0.0322</td>
<td>0.0964</td>
<td>0.0978</td>
<td>0.0247</td>
<td>0.0382</td>
</tr>
</tbody>
</table>
ment from the criteria, suitable normalization functions were developed. The method enabled selection with respect to the unit manufacturing cost and criteria of manufacturing quality, taking into consideration their importance, the optimal (the best) variant of the manufacturing process of the spindle with a collapsed balloon crown of a ring spinning frame. In particular, this procedure enables the best coupling of the method of finish machining preceding the surface treatment, which assures the highest hardness value of the surface layer after hard anodic oxidation at the allowable surface roughness and at relatively low manufacturing costs. The best variant of the manufacturing process of the neck of a spindle coating comprises the following final operations: finish turning with a diamond cutting edge and next hard anodic oxidation, followed by grinding with PS 20 corundum abrasive paper, grain size 600.

The implementation of this variant in industrial practice has enabled nearly fivefold growth of the operational lifetime comparing with that of the neck of the coating, which underwent operations of grinding with HTJ-13-3 corundum abrasive paper, grain size 150, and next grain size 320 (without anodic oxidation).

Very small cutting forces acting during finish turning, with the use of a diamond cutting edge, of a spindle neck made from AlCu4Mg1 alloy, not exceeding a value of 30 N, fully assure the obtaining of a radial run-out of the spindle neck, directly underneath the spindle crown, considerably below the allowable value. In manufacturing conditions it could have substantial importance for the quality of the yarn produced and for minimisation of the number of end breaks of the yarn (manufacturing capacity of the spinning frame).

References


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