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The Geometry and Kinematics of a Toothed Gear of Variable Motion

Abstract
The paper presents a toothed gear, in which an abaxially mounted toothed wheel is the active link, while a non-circular pinion (guaranteeing continuous interaction at a constant distance between the axes of rotation) is the passive link. Equations describing the shape of the rolling line of a non-circular pinion and its basic kinematic parameters have been determined.

Key words: *non-circular gear, geometry, kinematics.*

Introduction

Among many mechanisms transferring the drive from the active link to the passive, there are those whose function is to transform uniform rotary motion into continuous variable rotary motion. The demand for mechanisms offering the variable motion of the driven link was particularly visible in the construction of textile machines.

The drive of the slay in a loom (shown in the schematic diagram in Figure 1) which makes use of a pair of elliptical pinions resulted in an increase in the force of the weft beat-up in a fabric.

Pinions of this type are found in textile machines such as wool combing machines, presses for increasing the density of the materials pressed, cutters, and many others. In all these devices, elliptical pinions were nearly always used considering the necessity for continuous motion, and thoroughly investigated in respect of theory. All the mathematical relationships describing the geometry and kinematics of a gear with elliptical pinions are expressed by means of elementary functions, and hence any detailed analysis of them was impossible when the computer calculation technique was not known. The above problem was the subject of a doctoral dissertation [5] in which, besides the geometry and kinematics of elliptical gears, an attempt at designing such a gear for some precise kinematic requirements was presented. The author of this work, while dealing with the above theoretical issues, took interest in a different set of pinions, in which a circular pinion with an abaxial axis of rotation constituted an active link. One aspect was to make and analyse a new type of gear of variable transmission, but another was to find an answer to the following question: is the interaction of two identical toothed gears of the same abaxiality possible, when there is the same distance between the axes of rotation? Then, if it is possible, the question follows of what magnitude of abaxiality will ensure the correct interaction of the gears. In university textbooks [1,3], brief reports have appeared on such gears used in textile machine drives. The analysis of a toothed gear with an active circular pinion became possible when the computer calculation technique was introduced. The degree of complexity of the mathematical relationships describing the geometry and the kinematics of the above gear made it practically impossible to investigate these relationships without a computer.

Equations of Rolling Lines of Pinions

We have a circle of a radius a and an axis of rotation O_1, displaced with respect to the geometrical centre S by an abaxiality e. This circle is a rolling line of the active pinion P_1 of a toothed gear of variable transmission, as shown in Figure 2. The passive pinion P_2 has its rolling line in the form of a closed curve guaranteeing the rolling of these lines over each other, at a constant distance of the axis of rotation.

In the analysis, it is necessary to determine the radius r_1 as a function of the angle of rotation φ_1, and then the geometric and kinematic parameters of the interacting pinion. Based on the condition of the constant distance between the axes of rotation and the condition of rolling of the rolling lines, we obtained:

Function of ratio

$$\upsilon_{12}(\varphi) = \omega_1/\omega_2 = r_2(\varphi_1)/r_1(\varphi_1) \qquad (1)$$

Applying the geometric relationships of Figure 2 in (1):

$$r_1(\varphi) = - e \cos\varphi_1 + \sqrt{a^2 - e^2 \sin^2\varphi_1}$$

and $A = r_1(\varphi_1) + r_2(\varphi_1) = \text{const}$, we obtained:

$$\upsilon_{12}(\varphi_1) = (A_p - \rho(\varphi_1))/\rho(\varphi_1) \qquad (2)$$

where:

$$\rho(\varphi_1) = r_1(\varphi_1)/a = - e_p \cos \varphi_1 + \sqrt{1 - e_p^2 \sin^2 \varphi_1} \qquad (3)$$

is a dimensionless radius of the rolling line of the active pinion, $e_p=e/a$ is a dimensionless abaxiality and $A_p=A/a$ is a coefficient, referred to further as a dimensionless distance between the axes of rotation of the gear pinions.

Angular path of the passive pinion

The assumption of the rolling of the gear rolling lines without slide involves the following relationship:

$$d\varphi_2 = r_1(\varphi_1)/r_2(\varphi_1)\, d\varphi_1 \qquad (4)$$

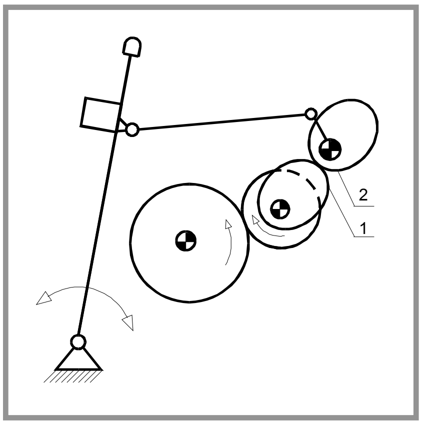

***Figure 1.** A schematic diagram of the slay drive acc. to Hartley and Farrar, with a variable transmission gear: 1 - the elliptical active pinion, 2 - the elliptical passive pinion.*

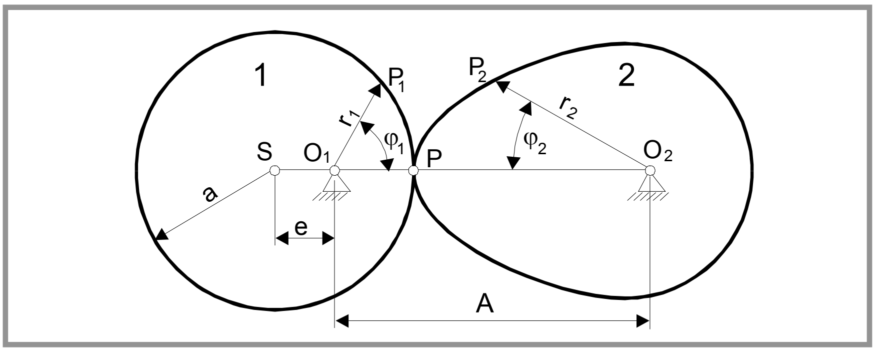

***Figure 2.** Rolling lines of the pinions of an outer abaxial gear (all designations are described in the chapter 'Equations of Rolling Lines of Pinions').*

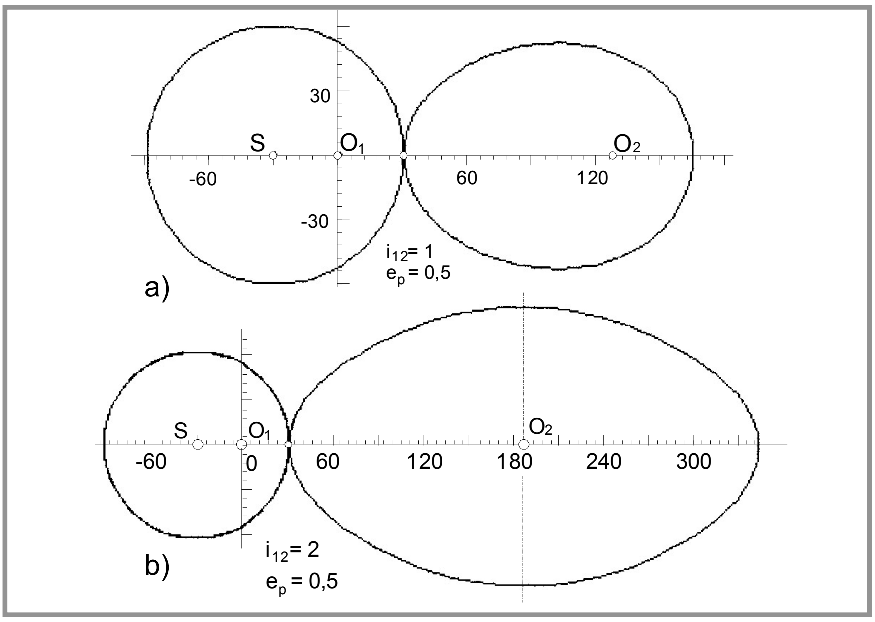

***Figure 3**. Rolling lines of the pinions of a gear for a=60, e_p=0.5 and a) i_{12}=1, b) i_{12}=2.*

Applying relationships (1) and (2) in (4), and after integration, we obtained:

$$\varphi_2=\int_0^{\varphi_1}\frac{\rho(\varphi_1)}{A_p-\rho(\varphi_1)}d\varphi_1 \qquad (5)$$

The integration constant was determined from the condition that for φ_1=0 there must be φ_2=0. Relationship (5) allows us to determine the angular path of the passive pinion for the assumed geometric parameters of the gears *a* and *e*. The third parameter, namely the distance A_p, necessary to solve equation (5), should be determined from the condition of closure of the rolling line of the passive pinion. With a mean ratio of the gear $i_{12}=n$, where: $n \in N$, this condition takes the form:

$$\varphi_2(2\pi)=2\pi=\int_0^{2\pi n}\frac{\rho(\varphi_1)}{A_p-\rho(\varphi_1)}d\varphi_1 \qquad (6)$$

Equation (6) was solved numerically considering the unknown value of A_p. For the assumed values of mean ratios i_{12}, we substituted successive values of e_p from the interval <0,1), obtaining equations with the unknown A_p. The solution to these equations leads to the determination of the function $A_p=A_p(e_p)$. Knowing the function $A_p(e_p)$, we can (for any assumed geometric parameters of the rolling line of the active pinion) determine the geometric parameters of the rolling line of the passive pinion. The solution to equation (5) is shown graphically in Figure 4.

Equation of the rolling line of the passive pinion

In the polar coordinates, this equation has the form $r_2=r_2(\varphi_2)$ and was expressed in the parametric form:

$$\begin{aligned} r_2(\varphi_1) &= a(A_p-\rho(\varphi_1)) \\ \varphi_2(\varphi_1) &= \int_0^{\varphi_1}\frac{\rho(\varphi)_1}{A_p-\rho(\varphi_1)}d\varphi_1 \end{aligned} \qquad (7)$$

It should be noted that, according to (6), if the closed rolling line of the passive pinion is to be determined, the angle φ_1 must change within the interval of 0 - $2\pi i_{12}$. Figure 3 shows example rolling lines of the gears for the assumed values of *a* and e_p, , as determined by means of a computer.

Path, Velocity and Angular Acceleration of the Passive Pinion

The function of the angular path of the passive pinion $\varphi_2(\varphi_1)$ is determined, as given above, by relationship (5). For the assumed mean ratio i_{12}=1 and selected values e_p=0-0.5, the angular paths are shown in Figure 4.

The function of the angular velocity of the passive pinion is determined from relationship (1):

$$\omega_2(\varphi) = \omega_1/\upsilon_{12}(\varphi_1) \qquad (8)$$

Applying relationships (2) and (3) in (8), after transformations, we will obtain:

$$\omega_2(\varphi_1)=\omega_1\frac{-e_p\cos\varphi_1+\sqrt{1-e_p^2\sin^2\varphi_1}}{A_p+e_p\cos\varphi-\sqrt{1-e_p^2\sin^2\varphi_1}} \qquad (9)$$

Figure 5 shows courses of the function of the angular velocity of the passive pinion, for i_{12}=1 and ω_1=1 s^{-1}, for the established values of e_p. For mean ratios i_{12}>1, the course of the function $\omega_2(\varphi_1)$ has a similar character. A comparison of the extreme values indicates that, with an increase in the mean ratio, the amplitude of changes in the velocity ω_2 decreases; hence the motion of the pinion, with the same abaxiality, becomes more uniform for higher values of i_{12}. Knowing that the function of angular acceleration of the passive pinion $\varepsilon_2(\varphi_1)$ is obtained by the differentiation of the function of angular velocity $\omega_2(\varphi_1)$, we have:

$$\varepsilon_2(\varphi_1)=\frac{d\omega_2}{dt}=\frac{d\omega_2}{d\varphi_1}\frac{d\varphi_1}{dt}=\omega_1^2\frac{d}{d\varphi_1}\left(\frac{1}{i_{12}}\right)$$

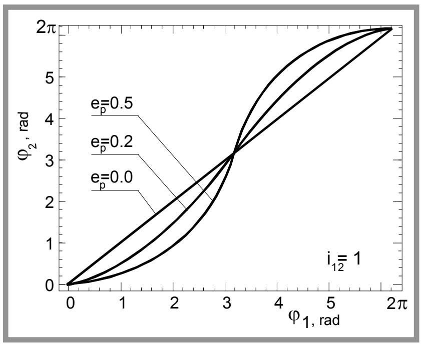

***Figure 4.** Angular path of the passive pinion.*

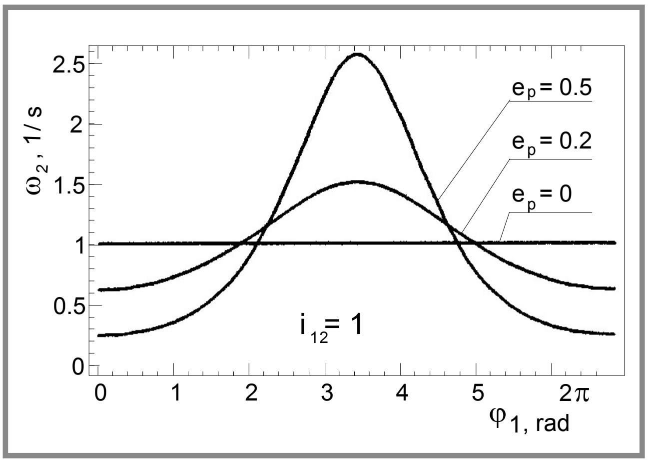

Figure 5. *Angular velocity of the passive pinion.*

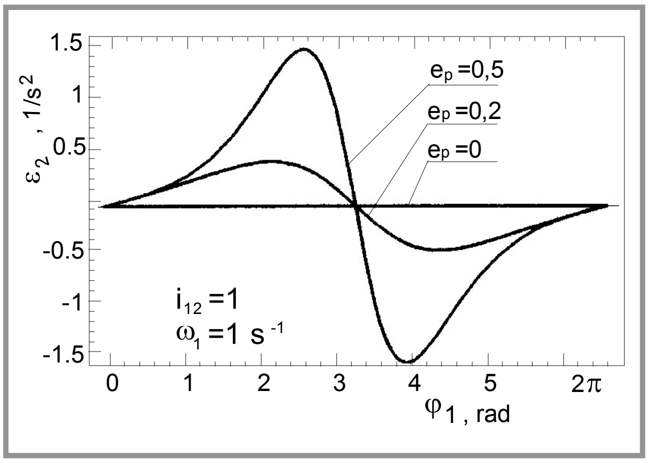

Figure 6. *Angular acceleration of the passive pinion.*

$$\varepsilon_2(\varphi_1)=A_p\omega_1^2 e_p \sin\varphi_1 \frac{e_p\cos\varphi+\sqrt{1-e_p^2\sin^2\varphi_1}}{\sqrt{1-e_p^2\sin^2\varphi_1}\;\left(A_p+e_p\cos\varphi_1-\sqrt{1-e_p^2\sin^2\varphi_1}\;\right)^2} \qquad (10)$$

Equation 10

After differentiation and transformations, we obtained equation 10.

A graphic image of the function of angular acceleration of the passive pinion is shown in Figure 6.

Conclusions

- The courses of velocity and acceleration of the passive pinion presented in the paper indicate that the required amplitude of changes in these functions can be obtained by selecting an appropriate value of the abaxiality e_p.
- For the established values of the abaxiality e_p and the mean ratio of the gear i_{12}, a dimensionless distance of the axes of rotation of the gear pinions A_p should be determined. The value of A_p must be calculated accurately to ensure the closing of the rolling line of the passive pinion.
- The analysis of the extreme velocities and accelerations of the passive pinion, with a constant abaxiality e_p, allows us to state that in gears with the mean ratio i_{12}=1, there is the greatest amplitude of changes in their courses. This gear is characterised by the greatest variation of motion.
- Special attention should be drawn to a non-circular pinion for a gear of the ratio i_{12}=2. Its characteristic feature is the preservation of the constancy of the function $A_p(e_p)$=2, over practically the whole range of changes in e_p. Geometrically and kinematicallyspeaking, a gear with this pinion is the most similar to a gear with elliptical pinions.
- The interaction of two identical toothed wheels with abaxially situated centres of rotation is impossible. Different, non-circular rolling lines are assigned to a wheel of different values of abaxiality, and only these ensure accurate fulfilment of the operating conditions of toothed gears.

References

1. *Chwalibóg M.: Kinematyka Mechanizmów Włókienniczych. T. 2, PWN, Łódź 1964.*
2. *Kowalczyk L., Urbanek S.: 'Design of the shape of the rolling lines of the pinions of a gear with variable motion', Scientific Letters of the Technical University of Łódź No. 760, Textile Science, z. 54, p. 25-34, 1996 (in Polish).*
3. *Małyszew A. P., Worobiew P. A.: Mechanika i konstruktywnyje razcziety tkackich stankow. Maszgiz 1960.*
4. *Ochęduszko K.: Koła zębate. T. 1. Konstrukcja. WNT. Warszawa 1985.*
5. *Urbanek S.: 'Geometrical analysis of the rolling lines of non-circular pinions of the elliptical type with consideration of selected kinematic and design demands', Technical University of Łódź, 1967, Ph.D. Dissertation (in Polish).*
6. *Zajączkowski J.: 'Mechanics of Textile Machines, Computer programme set', Technical University of Łódź, Department of Mechanics of Textile Machines (in Polish).*

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