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Applying Brent's Method for Calculating the Forces Acting on Sewing Manipulators

Abstract
The problem of calculating the driving and reaction forces which act on a sewing manipulator is studied in this paper. It is found that Brent's method is computationally efficient and can be applied for this purpose.

Key words: *sewing, manipulator, driving forces, Brent's method.*

Introduction

The process of sewing requires the following operations to be carried out: loading garment sections from stacks, folding, aligning edges, moving and joining with a thread while maintaining their mutual orientation. All these operations can be performed with manipulators equipped with suitable grippers and a needle-hook assembly.

A system composed of one manipulator equipped with a C-shaped end effector that carries a needle in the upper part and a hook in the lower part has been presented in work [1]. A system composed of two manipulators, one carrying a needle mechanism and the other carrying a hook mechanism, has been proposed in work [2]. A manipulator capable of gripping and removing a limp fabric from a stack and transporting it to a stitching apparatus has been proposed in work [3]. The sewing process can be performed on condition that the manipulator follows a desired trajectory. This requires an appropriate adjustment of driving forces during the motion. In the case of fabric sewing, serious difficulties arise from fabric limpness. The problem of calculating the forces acting on manipulators has been examined by a number of investigators [4,5]. The purpose of this work is to study a general numerical approach (based on Brent's method [6]) that will be suitable for any manipulator which can be used for sewing. Needle-hook and feed assemblies are not studied here. Grippers for textiles are considered in works [7,8]. Detailed discussions of applying manipulators for garment production can be found in works [9-11].

Forces Acting on the Manipulator

A schematic diagram of a general manipulator is shown in Figure 1. It consists of several rigid bodies mutually connected by a series of either rotary or sliding joints. One end of the chain is connected in this manner to a base, while the other end can move freely.

General rectangular co-ordinate frames, shown in Figure 1, are used in this paper. If the neighbouring co-ordinate systems w have one parallel axis of the same name, then the transformation from one system to the other requires only the translation or the rotation about the parallel axis. The parallel axis condition can be met by taking a sufficient number of co-ordinate systems and by choosing their placement correctly. The co-ordinates of point P in the system w_n are x_n, y_n, z_n. The desired trajectory of point P is defined by the following time functions:

$$\begin{aligned} x_0^{(r)} &= x(t), \\ y_0^{(r)} &= y(t), \\ z_0^{(r)} &= z(t), \\ \angle(x_0, x_n)^{(r)} &= \varphi_{x_0,x_n}(t), \\ \angle(y_0, y_n)^{(r)} &= \varphi_{y_0,y_n}(t) \end{aligned} \tag{1}$$

In order to calculate inertia forces, the mass of the manipulator is divided into parts (circles in Figure 2), concentrated at the origins of the co-ordinate systems. The force acting on the mass k is given by

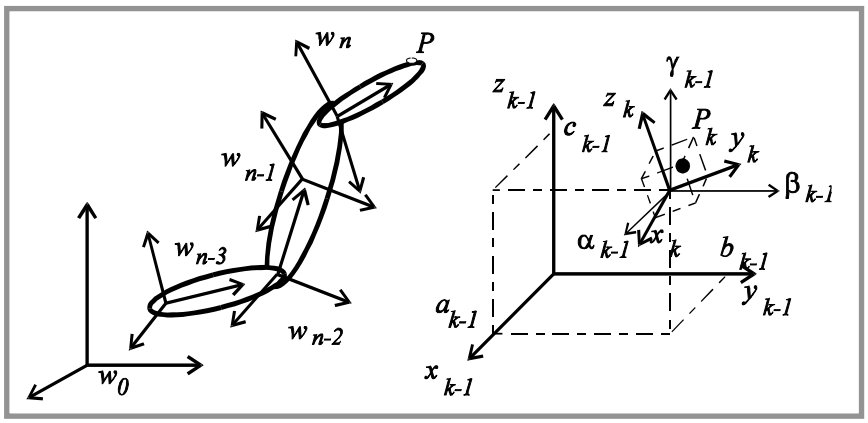


Figure 1. *Manipulator and the co-ordinate system.*

$$F_k^{(w_0)} = -m_k \frac{\mathrm{d}^2 w_0^{(k)}}{\mathrm{d}t^2} + F_{ke}^{(w_0)}$$

$$F = \begin{bmatrix} F^{(x)} \\ F^{(y)} \\ F^{(z)} \end{bmatrix} \tag{2}$$

The superscript (w_0) is used to indicate that the force is given by its components in the system w_0. The moment of the force k about point l is given by its components

$$M_{k,l} = \begin{bmatrix} r_{k,l}^{(y)} F_k^{(z)} - r_{k,l}^{(z)} F_k^{(y)} \\ r_{k,l}^{(z)} F_k^{(x)} - r_{k,l}^{(x)} F_k^{(z)} \\ r_{k,l}^{x} F_k^{(y)} - r_{k,l}^{(y)} F_k^{(x)} \end{bmatrix}$$

$$r_{k,l} = w_0^{(k)} - w_0^{(l)}, \quad M = \begin{bmatrix} M^{(x)} \\ M^{(y)} \\ M^{(z)} \end{bmatrix} \tag{3}$$

A summation of the forces on free side of the joint l gives the resultant force acting on that joint:

$$F_{sl}^{(w_0)} = F_l^{(w_0)} + F_{l+1}^{(w_0)} + F_{l+2}^{(w_0)} + \ldots \tag{4}$$

A summation of the moments of the forces about the joint l gives the resultant moment about the joint l:

$$M_{sl}^{(w_0)} = M_{l+1,l}^{(w_0)} + M_{l+2,l}^{(w_0)} + \\ + M_{l+3,l}^{(w_0)} + M_{l+4,l}^{(w_0)} + \ldots \tag{5}$$

The joint forces (4) and the joint moments (5) are defined by their components in the system w_0. Their components in the joint co-ordinate system w_l are the driving and reaction forces and moments.

The dependence between the rectangular components of the vectors in co-ordinate system w_k and w_{k-1}, shown in Figure 2 is given by

$$F^{(w_k)} = Q_{k-1} F^{(w_{k-1})} \\ M^{(w_k)} = Q_{k-1} M^{(w_{k-1})} \tag{6}$$

The matrices Q for the co-ordinate system w_{n-1} rotated about x, y and z axis are given respectively by

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$Q = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \tag{7}$$

$$Q = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

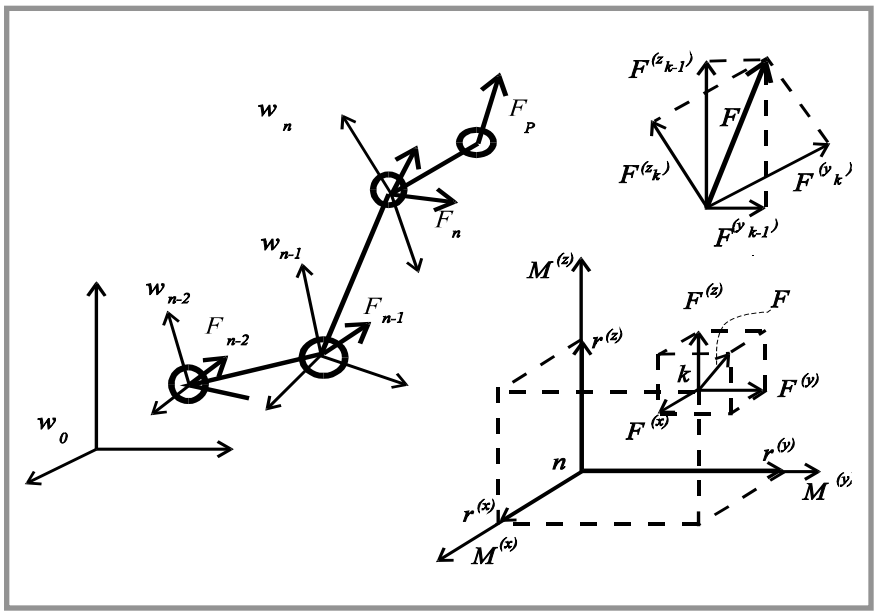


Figure 2. *The forces acting on the manipulator.*

$$\frac{dw}{dt}(t_n) = \frac{1}{12\Delta t}\left(-25W_n + 48W_{n+1} - 36W_{n+2} + 16W_{n+3} - 3W_{n+4}\right)$$

$$\frac{d^2 w}{dt^2}(t_n) = \frac{1}{24\Delta t^2}\left(70W_n - 208W_{n+1} + 228W_{n+2} - 112W_{n+3} + 22W_{n+4}\right)$$

$$\frac{dw}{dt}(t_n) = \frac{1}{12\Delta t}\left(W_{n-2} - 8W_{n-1} + 8W_{n+1} - W_{n+2}\right)$$

$$\frac{d^2 w}{dt^2}(t_n) = \frac{1}{24\Delta t^2}\left(-2W_{n-2} + 32W_{n-1} - 60W_n + 32W_{n+1} - 2W_{n+2}\right)$$

$$\frac{dw}{dt}(t_n) = \frac{1}{12\Delta t}\left(3W_{n-4} - 16W_{n-3} + 36W_{n-2} - 48W_{n-1} + 25W_n\right)$$

$$\frac{d^2 w}{dt^2}(t_n) = \frac{1}{24\Delta t^2}\left(22W_{n-4} - 112W_{n-3} + 228W_{n-2} - 208W_{n-1} + 70W_n\right)$$

where:

$W_n = w(t_n)$

$w = w_0^{(k)}$

$t_{n+1} = t_n + \Delta t$.

Equation 10

Repeated back-substitution of the equations (6) gives the components of the resultant force and moment in the co-ordinate system of a joint l.

$$F_{sl}^{(w_l)} = Q_{l-1} Q_{l-2} \ldots Q_1 Q_0 F_{sl}^{(w_0)} \tag{8}$$

$$M_{sl}^{(w_l)} = Q_{l-1} Q_{l-2} \ldots Q_1 Q_0 M_{sl}^{(w_0)} \tag{9}$$

With the components of vectors (4,5) in the co-ordinate system w_0, one can find their components in the joint co-ordinate system w_l using formulas (8,9). The velocity and acceleration of the mass k can be found using the formulas for numerical differentiation presented in equation (10).

The first two equations (10) are needed at the beginning of the motion, while the last equations are needed at the end of the motion. With the co-ordinates of the mass in the system k, the co-ordinates in the system 0 are found from

$$\begin{bmatrix} w_0^{(k)} \\ 1 \end{bmatrix} = T_0 T_1 T_2 \ldots T_{k-1} \begin{bmatrix} w_k \\ 1 \end{bmatrix}$$

$$w = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{11}$$

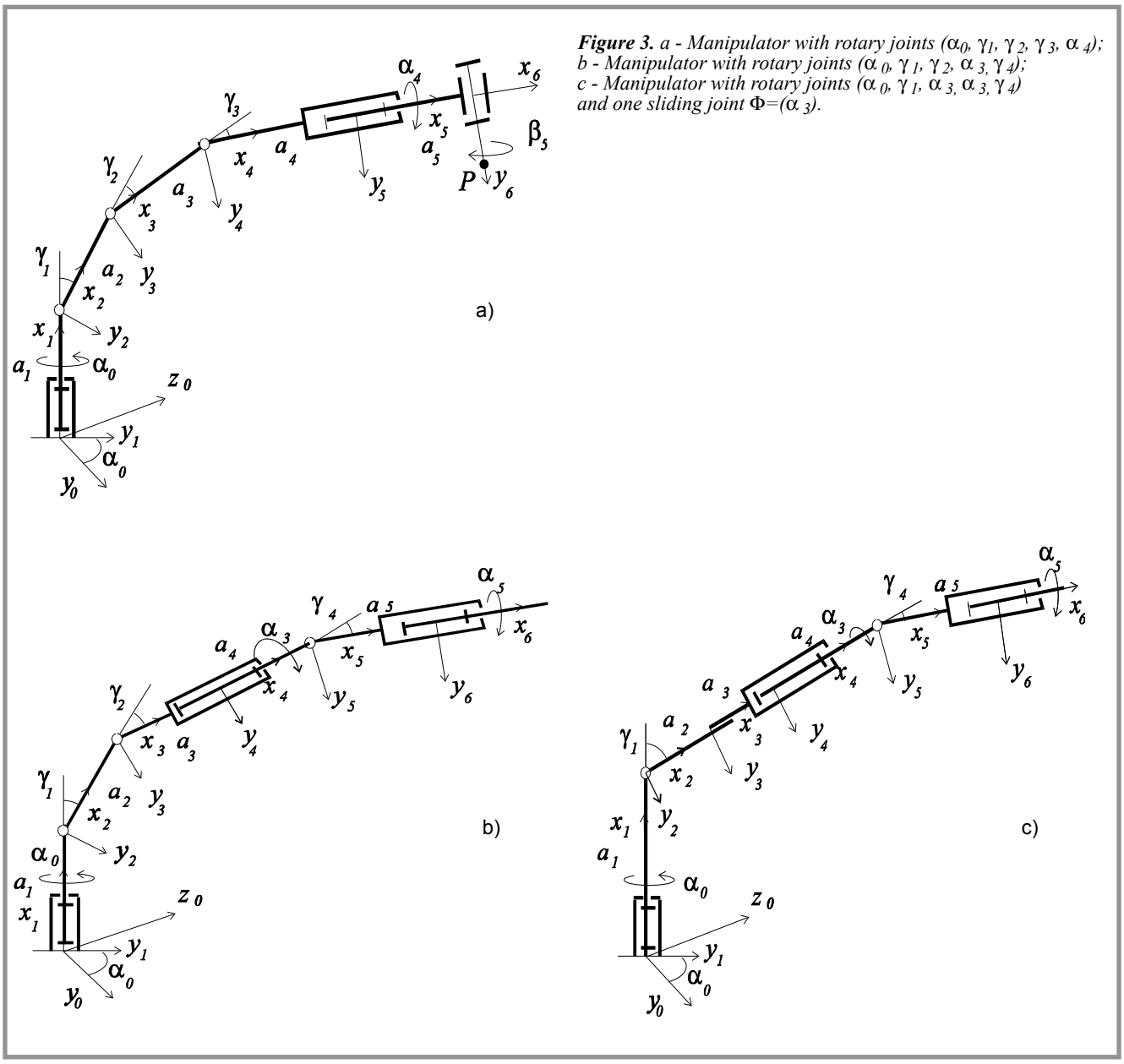
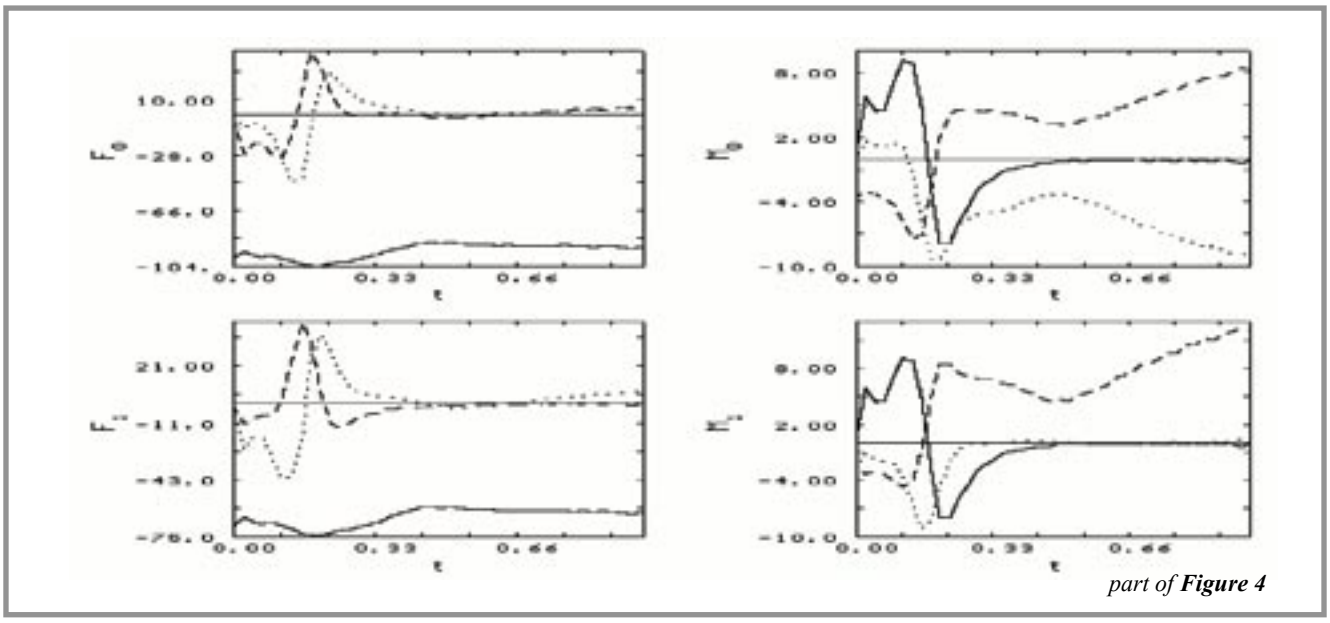


Figure 3. *a - Manipulator with rotary joints (α_0, γ_1, γ_2, γ_3, α_4); b - Manipulator with rotary joints (α_0, γ_1, γ_2, α_3, γ_4); c - Manipulator with rotary joints (α_0, γ_1, α_3, α_3, γ_4) and one sliding joint Φ=(α_3).*



part of ***Figure 4***

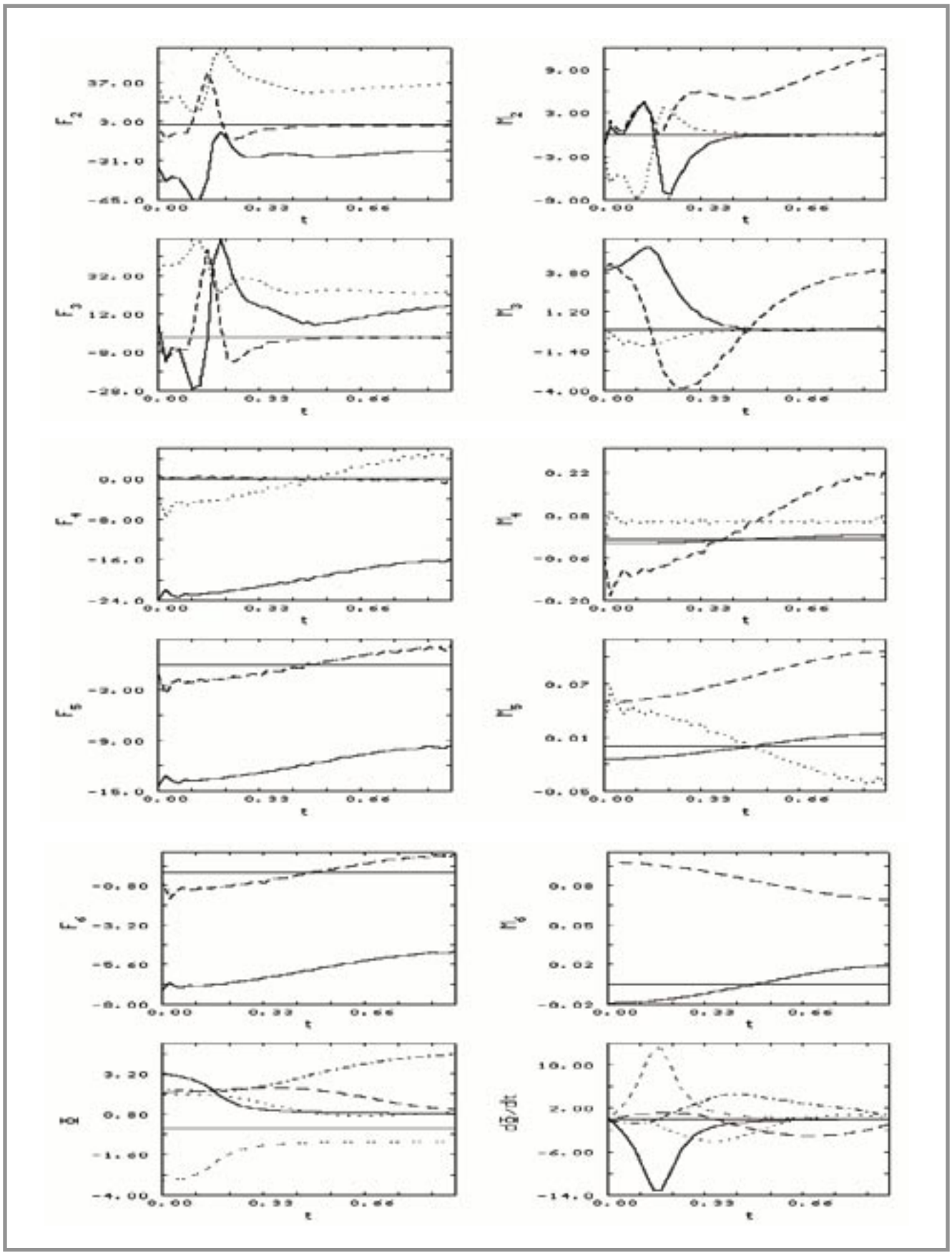


Figure 4. *Components of driving and reaction forces* $F_j = F_{ij}^{(w_j)}$ *[N] and moments* $M_j = M_{ij}^{(w_j)}$ *[Nm] (for* F_0, F_1...F_6 *and* M_0, M_1...M_6*) in the x-y-z co-ordinate systems* w_j *versus time t [s]. For each force, the component curves have been drawn by the following lines:* $F^{(x)}$, $M^{(x)}$ *(——),* $F^{(y)}$, $M^{(y)}$*: (·····) ,* $F^{(z)}$, $M^{(z)}$*: (– – –); relative positions* $\Phi = (\alpha_0, \gamma_1, \gamma_2, \gamma_3, \alpha_4)$ *[rad] and relative velocities* $d\Phi / dt = (d\alpha_0 / dt, d\gamma_1 / dt, d\gamma_2 / dt, d\gamma_3 / dt, d\alpha_4 / dt)$ *[rad/s] of system elements;* α_0, $d\alpha_0/dt$*: (——),* γ_1, $d\gamma_1/dt$*: (·····),* γ_2, $d\gamma_2/dt$*: (– – –),* γ_3, $d\gamma_3/dt$*: (– · – · –),* α_4, $d\alpha_4/dt$*: (·· ·· ··).*

Here, the matrices T for the rotations about x, y and z axis are given respectively by

$$T = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & \cos\alpha & -\sin\alpha & b \\ 0 & \sin\alpha & \cos\alpha & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} \cos\beta & 0 & \sin\beta & a \\ 0 & 1 & 0 & b \\ -\sin\beta & 0 & \cos\beta & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (12)$$

$$T = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 & a \\ \sin\gamma & \cos\gamma & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For the desired co-ordinates of the point P in the system w_0 and the desired angles $\angle(x_0, x_n)$, $\angle(y_0, y_n)$ the vector

$$\Phi = (a_i, b_j, c_k, \alpha_l, \beta_m, \gamma_n),$$

defining the relative positions of the system elements at joints, can be found from the following equation:

$$f(\Phi) = \begin{bmatrix} x_0^{(r)} - x_0 \\ y_0^{(r)} - y_0 \\ z_0^{(r)} - z_0 \\ \Gamma_{[n,n]}^{(r)} - \Gamma_{[n,n]} \\ \Gamma_{[m,m]}^{(r)} - \Gamma_{[m,m]} \end{bmatrix} = 0 \qquad (13)$$

Here, superscript (r) denotes the required co-ordinates. The actual co-ordinates corresponding to the vector Φ are given by equations (14) and (15).

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} = T_0 T_1 T_2 \ldots T_{n-1} \begin{bmatrix} x_n \\ y_n \\ z_n \\ 1 \end{bmatrix} \qquad (14)$$

With the p-th approximation of the solution Φ of equation (13), one can find the next approximation as

$$\Phi_{p+1} = \Phi_p + \delta\Phi \qquad (16)$$

where the vector $\delta\Phi$ is found from the linear algebraic equation

$$\frac{\partial f}{\partial \Phi}(\Phi_p)\delta\Phi = -f(\Phi_p) \qquad (17)$$

Here, $\partial f / \partial \Phi$ is a Jacobi matrix. The calculations must be repeated until the value of f found from (13) and the value of $\delta\Phi$ found from (17) become sufficiently small. In this paper the calculations were carried out using Brent's method [6].

Numerical Results

The algorithm described in this paper can be applied to any manipulator. Example calculations were carried out for three different manipulators, which are shown in Figures 3a, 3b, 3c. For manipulator (3a) the vector defining the relative positions of the system elements is

$$\Phi = (\alpha_0, \gamma_1, \gamma_2, \gamma_3, \alpha_4),$$

for manipulator (3b) it is

$$\Phi = (\alpha_0, \gamma_1, \gamma_2, \alpha_3, \gamma_4),$$

and for manipulator (3c) it is

$$\Phi = (\alpha_0, \gamma_1, \alpha_3, \alpha_3, \gamma_4).$$

The method was found to work for all of them.

Only the example results for manipulator shown in Figure 3a are presented here. The dimensions defined in Figure 1 and shown in Figure 3a were taken as $a=b=c=0$ with the exception of $a_1=0.2m$, $a_2=0.2\sqrt{2}m$, $a_3=0.2\sqrt{2}m$, $a_4=0.04m$, $a_5=0.04m$: the angles were $\alpha=\beta=\gamma=0$ apart from the vector defining the relative positions of the system elements ($\Phi=\alpha_0,\gamma_1,\gamma_2,\gamma_3,\alpha_4$). The manipulator thus defined is shown in Figure 3a.

The masses m_n [kg] (circles in Figure 2) concentrated at the origins of the co-ordinate systems (x_n, y_n, z_n in Figure 3a), were taken as $m_0=3$, $m_1=2$, $m_2=1.5$, $m_3=1$, $m_4=0.8$, $m_5=0.6$ and $m_6=0.4$; the mass located at point P was taken as $m_p=0.2$.
The co-ordinates of the point P in the system w_6 were taken as $x_6=0$, $y_6=0.04$m, $z_6=0$; the co-ordinates [m] of that point in the system w_0 are defined as

$$x_0 = 0.36\xi(t)$$
$$y_0 = 0.36\eta(t)$$
$$z_0 = 0.36\zeta(t)$$
$$\angle(x_0, x_6) = 0, \angle(y_0, y_6) = 0$$

where

$$\xi(t) = \eta(t) = \zeta(t) = 1/2(1-\cos(\pi t/T_m))$$

The components of the driving and reaction forces $F_j = F_{ij}^{(w_j)}$ and the moments $M_j = M_{ij}^{(w_j)}$ in co-ordinate systems w_l, the relative position vector

$$\Phi=\alpha_0,\gamma_1,\gamma_2,\gamma_3,\alpha_4$$

and the relative velocities

$$d\Phi/dt = d(\alpha_0,\gamma_1,\gamma_2,\gamma_3,\alpha_4)$$

of system elements calculated for $T_m=1s$ and $\Delta t = T_m/45$ are shown in Figure 4.

Conclusion

The computations have shown that Brent's method is computationally efficient and can be applied for calculating the forces acting on a manipulator.

$$\Gamma = \begin{bmatrix} \cos(x_0, x_n) & \cos(x_0, y_n) & \cos(x_0, z_n) & a_{0,n} \\ \cos(y_0, x_n) & \cos(y_0, y_n) & \cos(y_0, z_n) & b_{0,n} \\ \cos(z_0, x_n) & \cos(z_0, y_n) & \cos(z_0, z_n) & c_{0,n} \\ 0 & 0 & 0 & 1 \end{bmatrix} = T_0 T_1 T_2 \ldots T_{n-1}$$

Equation 15

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