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Mathematical Model of a Mechatronic Parallel Finger Gripper for Textiles

Abstract
This paper presents the elaboration of a mathematical model for a computer-controlled gripper with parallel fingers for handling textile fabric pieces. The description of this model includes the dynamic equations of the gripper movements, the equations for the magnetic moments of the driving motor, and the relations between the forces and deformations of the textile material. The obtained set of equations is solved numerically and the results are plotted.

Key words: *gripper for textiles, computer control, mathematical model, garment production, robotisation.*

Introduction

A discussion of papers and patents concerned with grippers for textiles is presented in the author's previous work [2]. A mathematical model of a pinch-type gripper is presented therein. The gripper contacts the fabrics on a very small, point-like surface. As a result, the required force may produce too high a pressure, which could damage the fabric. The problem may be solved by the utilisation of grippers with parallel fingers, where the contacting surface can be sufficiently large. In this work, a mathematical model of such a gripper is developed.

Parallel Finger-type Gripper

The gripper discussed, and presented in Figure 1, is characterised by parallel fingers. The device is driven with the use of an electric motor by means of a screw of pitch h_S. The rotating screw forces a longitudinal movement of the gripping device elements, and forces the closing and opening of the gripper. It should be emphasised that the gripper fingers also move along a straight line.

The mathematical model was elaborated in accordance with the procedure described in [1]. The equations describing relations between co-ordinates are found in the following form:

$$x = x_0 - \frac{\varphi}{2\pi} h_S \tag{1}$$

$$F_1 = r_5 \cos \alpha_1 + r_6 \cos \alpha_6 - (r_7 - x) = 0$$

$$F_2 = r_5 \sin \alpha_1 - r_6 \sin \alpha_6 = 0$$

The dynamic equation of movement (2) can be presented using the principle of

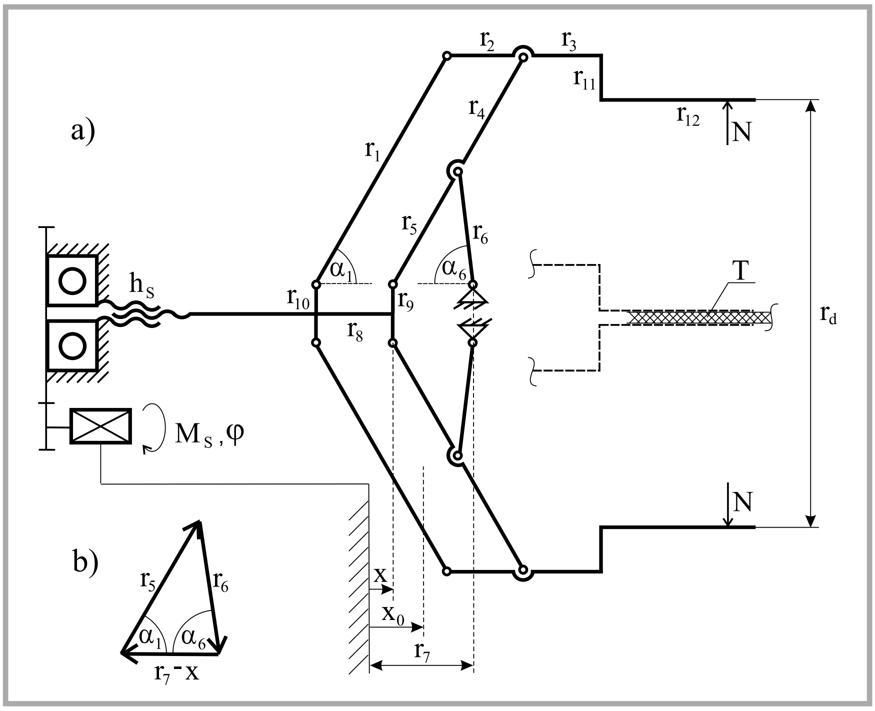

***Figure 1.** a) Diagram of the gripper; b) Closed vector polygon. Denotations: M_S - motor driving moment, φ - rotation angle of the main shaft, h_S - screw pinch, r_1... r_{12} - lengths of the gripper elements, α_1, α_6 - angles between a given gripper element and the horizontal axis, x_0 - initial position of the gripper, x - position of the gripper at a given time, N - force compressing the material, T - woven fabric, r_d - distance between the fingers at a given moment.*

$$M_S - I_S\frac{d^2\varphi}{dt^2} - 2m_3\frac{d^2y}{dt^2}\frac{dy}{d\varphi} - m_8\frac{d^2x}{dt^2}\frac{dx}{d\varphi} - 2I_1\frac{d^2\alpha_1}{dt^2}\frac{d\alpha_1}{d\varphi} - 2I_4\frac{d^2\alpha_1}{dt^2}\frac{d\alpha_1}{d\varphi} - 2I_6\frac{d^2\alpha_6}{dt^2}\frac{d\alpha_6}{d\varphi} + 2N\frac{dy}{d\varphi} - 2m_1\frac{d^2x_{C1}}{dt^2}\frac{dx_{C1}}{d\varphi} + $$

$$- 2m_4\frac{d^2x_{C4}}{dt^2}\frac{dx_{C4}}{d\varphi} - (2m_1 + 2m_4)\frac{d^2y_C}{dt^2}\frac{dy_C}{d\varphi} = 0 \tag{2}$$

$$\left[I_S + 2m_3\left(\frac{dy}{d\varphi}\right)^2 + m_8\left(\frac{dx}{d\varphi}\right)^2 + (2I_1 + 2I_4)\left(\frac{d\alpha_1}{d\varphi}\right)^2 + 2I_6\left(\frac{d\alpha_6}{d\varphi}\right)^2 + 2m_1\left(\frac{dx_{C1}}{d\varphi}\right)^2 + 2m_4\left(\frac{dx_{C4}}{d\varphi}\right)^2 + (2m_1 + 2m_4)\left(\frac{dy_C}{d\varphi}\right)^2\right]\frac{d^2\varphi}{dt^2} + $$

$$+ \left[2m_3\frac{d^2y}{d\varphi^2}\frac{dy}{d\varphi} + m_8\frac{d^2x}{d\varphi^2}\frac{dx}{d\varphi} + (2I_1 + 2I_4)\frac{d^2\alpha_1}{d\varphi^2}\frac{d\alpha_1}{d\varphi} + 2I_6\frac{d^2\alpha_6}{d\varphi^2}\frac{d\alpha_6}{d\varphi} + 2m_1\frac{d^2x_{C1}}{d\varphi^2}\frac{dx_{C1}}{d\varphi} + 2m_4\frac{dx_{C4}}{d\varphi^2}\frac{dx_{C4}}{d\varphi} + \right. \tag{8}$$

$$\left. + (2m_1 + 2m_4)\frac{d^2y_C}{d\varphi^2}\frac{dy_C}{d\varphi}\right]\left(\frac{d\varphi}{dt}\right)^2 - 2N\left(\frac{dy}{d\varphi}\right) - M_S = 0$$

$$\frac{d\varphi_1}{dt} = \varphi_2$$

$$\frac{d\varphi_2}{dt} = \frac{M_S + 2N\frac{dy}{d\varphi_1} - \left[2m_3\frac{d^2y}{d\varphi_1^2}\frac{dy}{d\varphi_1} + m_8\frac{d^2x}{d\varphi_1^2}\frac{dx}{d\varphi_1} + (2I_1 + 2I_4)\frac{d^2\alpha_1}{d\varphi_1^2}\frac{d\alpha_1}{d\varphi_1} + I_6\frac{d^2\alpha_6}{d\varphi_1^2}\frac{d\alpha_6}{d\varphi_1} + 2m_1\frac{d^2x_{C1}}{d\varphi_1^2}\frac{dx_{C1}}{d\varphi_1} + 2m_4\frac{d^2x_{C4}}{d\varphi_1^2}\frac{dx_{C4}}{d\varphi_1} + (2m_1 + 2m_4)\frac{d^2y_C}{d\varphi_1^2}\frac{dy_C}{d\varphi_1}\right]\left(\frac{d\varphi_1}{dt}\right)^2}{I_S + 2m_3\left(\frac{dy}{d\varphi_1}\right)^2 + 2m_8\left(\frac{dx}{d\varphi_1}\right)^2 + (2I_1 + 2I_4)\left(\frac{d\alpha_1}{d\varphi_1}\right)^2 + 2I_6\left(\frac{d\alpha_6}{d\varphi_1}\right)^2 + 2m_1\left(\frac{dx_{C1}}{d\varphi_1}\right)^2 + 2m_4\left(\frac{dx_{C4}}{d\varphi_1}\right)^2 + (2m_1 + 2m_4)\left(\frac{dy_C}{d\varphi_1}\right)^2} \tag{9}$$

Equations 2, 8, 9

virtual work, where: φ - the rotation angle of the main shaft, I_S – the moment of inertia of the main shaft, I_1, I_4, I_6 - the moments of inertia of the gripper elements with r_1, r_4, and r_6 length, m_1, m_3, m_4, m_8 – the masses of the corresponding gripper elements, r_3, r_{12} – the gripper finger lengths, α_1, α_6 – the corresponding angles, M_S – the driving torque of the motor, N – the compression force.

It is convenient to express the driving torque M_S in terms of mechanical quantities:

$$\frac{dM_S}{dt} = \frac{1}{T}\left[c\left(\Omega - \frac{d\varphi}{dt}\right) - M_S\right], \tag{3}$$

$$T = \frac{L}{R}, \quad c = \frac{K_t \cdot K_b}{R},$$

$$\Omega = \frac{e}{K_b}, \quad i = \frac{M_s}{Kt}$$

where:

- φ - the angle of revolution,
- t - time,
- T - the time constant of the motor,
- c - the stiffness of the motor characteristic,
- Ω - the angular velocity of the motor shaft at which the moment equals zero,
- L - inductance,
- R - resistance,
- e - supply voltage,
- i - the current intensity,
- K_b - the voltage constant,
- K_t - the turning moment constant.

The dependence of compression u on the force N is assumed to be of the form:

$$N = k_1u + k_2u^3 + c\frac{du}{dt} \quad \text{for } u>0$$

$$N = 0 \quad \text{for } u\le 0 \tag{4}$$

where:

- k_1, k_2 - elasticity constants,
- c - the coefficient of energy dissipation.

The total deformation u of the fabric stack can be expressed by the following relations:

$$u = g - r_d$$
$$r_d = 2r_{12} + 2r_1\sin\alpha_1 - 2r_{11} \ge 0 \tag{5}$$

$$\frac{du}{dt} = -\frac{dr_d}{dt} = -2r_1\cos\alpha_1\frac{d\alpha_1}{dt} \tag{6}$$

The normal force in the sample under consideration is the force which compresses the fabric under the condition of $r_d < g$, where g is the thickness of the fabric stuff. If $r_d \ge g$ then the force N=0.

Making use of the formula for derivative (7) and substituting into expression (2), the following equation (8) is obtained:

$$\frac{d\alpha}{dt} = \frac{d\alpha}{d\varphi}\frac{d\varphi}{dt}$$

$$\frac{d^2\alpha}{dt^2} = \frac{d^2\varphi}{dt^2}\frac{d\alpha}{d\varphi} + \left(\frac{d\varphi}{dt}\right)^2\frac{d^2\alpha}{d\varphi^2}$$

$$\frac{dy}{dt} = \frac{dy}{d\varphi}\frac{d\varphi}{dt} \tag{7}$$

$$\frac{d^2y}{dt^2} = \frac{d^2\varphi}{dt^2}\frac{dy}{d\varphi} + \left(\frac{d\varphi}{dt}\right)^2\frac{d^2y}{d\varphi^2}$$

$$\frac{dx}{dt} = \frac{dx}{d\varphi}\frac{d\varphi}{dt}$$

$$\frac{d^2x}{dt^2} = \frac{d^2\varphi}{dt^2}\frac{dx}{d\varphi} + \left(\frac{d\varphi}{dt}\right)^2\frac{d^2x}{d\varphi^2}$$

By denoting angular velocity by symbol φ_2, one may write the system of first-order equations in the form presented as (9).

The feedback was inserted in the computer program similar to the gripper analysed in [2], with this difference that the de value was calculated from the following equation:

$$de = e \cdot \left|\frac{N - N_{set}}{600000}\right| \tag{10}$$

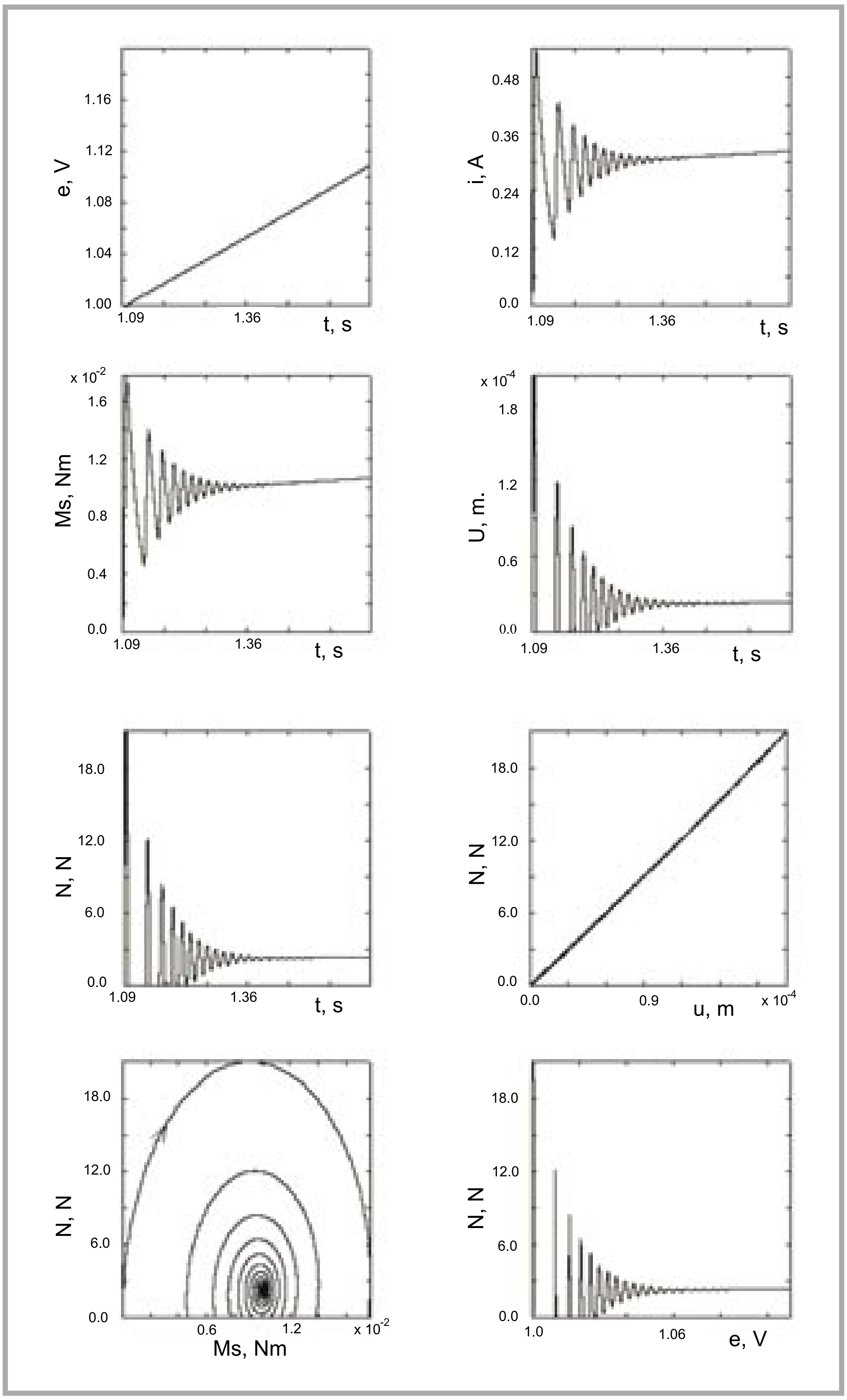

***Figure 2.** Time history of the electromotive force e [V], the electric current intensity i [A], the driving torque M_S [Nm], the fabric material deformation u [m] and the force compressing the fabric N [N]; the relationships between N and the following quantities: u, M_S, e.*

Results and Discussion

The computer program simulating the gripper, described by the above equations, was developed. The parameters suitable for handling a piece of fabric were chosen. The calculations were carried out for the following parameters:

- dimensions: r_1=0.05 m, r_2=0.015 m, r_3=0.01 m, r_4=0.025 m, r_5=0.025 m, r_6=0.025 m, r_7=0.015 m, r_8=0.015 m, r_9=0.005 m, r_{10}=0.005 m, r_{11}=0.009 m, r_{12}=0.03 m, x_0=0.012 m, h_S=0.001 m
- mass moments of inertia: I_1=178·10^{-9} kgm², I_4=178·10^{-9} kgm², I_6=22·10^{-9} kgm²
- mass of the corresponding elements of the gripper fingers: m_1=85·10^{-5} kg, m_3=110·10^{-5} kg, m_4=85·10^{-5} kg, m_8=60·10^{-5} kg.

An electric motor was characterised by a voltage constant K_b=0.038V/(rad·s), turning moment constant K_t=0.033 Nm/A, resistance R=3.43 Ω, inductance L=0.53·10^{-3} H, supply voltage e=100 V, and mass moment of inertia of the motor rotor I_S=1.0·10^{-5} kgm^2.

The results of the calculations are shown in Figure 2. The time histories of the automatically adjusted supply voltage, the electric current intensity, the driving torque, the fabric material deformation and the force compressing the fabric are plotted; the relationships between the acting force N and resulting contraction u of the fabric, and the dependence of the force on the supply voltage and driving torque are shown.

Conclusion

The computer program describing the gripper's work makes it possible to analyse the fabric material's reaction and enables the designer to choose appropriate parameters. The simulation of the gripper's work is especially important for preventing the occurrence of transient forces that are destructive to the fabric.

All quantities characterising the gripper behaviour achieved their limit values. Although the mathematical description of this gripper is partially different from that for the pinch-type gripper [2], the results are similar with the exception that the pressure necessary to hold up the fabric can be lower here.

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