Mechanics of Parallel Fibre Bundles

Abstract
This theoretical work deals with the mechanics of parallel fibre bundles, on the basis of the fact that each fibre in the bundle possesses different tensile behaviours. As a consequence of this, the tensile behaviour of a blended fibre bundle is found to be different than that obtained from Hamburger’s theory [3]. It is also observed that the average force per fibre in the bundle, the breaking force utilisation coefficient, and the breaking strain utilisation coefficient depend only on the coefficient of variation of the fibre breaking strain.

Key words: parallel fibre bundle, random fibre breaking points, similar force-strain relation, symmetrical breaking force, utilisation coefficient, coefficient of variation of fibre breaking strain.

Introduction
The tensile behaviour of parallel fibre bundles has always been an interesting topic for textile researchers. It is well known that the tensile properties of a fibre bundle are greatly influenced by the tensile properties of the constituent fibres which form the bundle. Therefore, a complete understanding of the mechanism of translation of stress-strain curves of the constituent fibres into the tensile properties of the bundle is of great importance. In this regard, perhaps the simplest theoretical model assumes that all of the constituent fibres of a fibre bundle follow the same stress-strain curve and have the same breaking stress and breaking strain. Modelling the tensile behaviour of such a bundle is a trivial task. The tensile properties of a multi-component fibre bundle, where all the components relate to the same fibre material, were first formulated by Sinitsin [1]; subsequently, those formulas were found to be in good agreement with the actual results of spun yarns produced from the mixing of different varieties of Egyptian cotton fibres of different lengths and fineness [2].

Assumption of similarity in force
S - strain relation of fibres
The fibres have a similar force-strain relation such that at or before a fibre breaks (σ ≤ α), its tensile behaviour follows the relation . As we call it, is an average function characterising the average force-strain relation of fibres, and k is a fibre param-
eter. Here we introduce the convention that the average function passes through the average breaking point of fibres, as shown in Figure 2. Hence the following expression is obvious:

\[ \bar{P} = \bar{S}(\bar{\alpha}) \]  

Thus the following relation holds at the breaking point of each fibre:

\[ P = S(\alpha) = k\bar{S}(\alpha), \text{ or } k = P/\bar{S}(\alpha) \]  

So the force-strain relation of a general fibre can be expressed as follows:

\[ S = S(\varepsilon) = k\bar{S}(\varepsilon) = [P/\bar{S}(\alpha)]\bar{S}(\varepsilon), \]  

when \( \varepsilon \leq \alpha \)

\[ S = 0, \text{ when } \varepsilon > \alpha \]  

The average force per fibre in the fibre bundle \( S^* \) is given by

\[ S^* = \frac{\int_{\alpha_{\min}}^{\alpha_{\max}} \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} S(P, \alpha) dP da}{\int_{\alpha_{\min}}^{\alpha_{\max}} \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} S(\alpha, \varepsilon) d\alpha d\varepsilon} \]  

On the basis of the above two assumptions, \( S^* \) takes the following forms.

Case 1 (no fibre is broken): substituting \( S \) from Equation (10a) into (11) and then utilising (7), we obtain

\[ S^* = \bar{S}(\varepsilon) \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{P(\alpha)}{\bar{S}(\alpha)} g(\alpha) d\alpha \]  

when \( \varepsilon < \alpha_{\min} \)

Case 2 (fibres with \( a < \varepsilon \) are broken): in analogy to the derivation of Equation (12a), we obtain

\[ S^* = \bar{S}(\varepsilon) \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{P(\alpha)}{\bar{S}(\alpha)} g(\alpha) d\alpha \]  

when \( \varepsilon \in (\alpha_{\min}, \alpha_{\max}) \)

Case 3 (all fibres are broken): then \( S = 0 \), hence obviously

\[ S^* = 0, \text{ when } \varepsilon > \alpha_{\max} \]  

It is also possible to derive an expression for the breaking force of the fibre bundle related to one fibre. This is the maximum of the average force per fibre in the fibre bundle. In this context, we consider the most common type of force-strain behaviour of a fibre bundle, as shown in Figure 3, with the breaking force of the bundle related to one fibre \( P^* \) and the breaking strain of the bundle related to one fibre \( \varepsilon^* \in (\alpha_{\min}, \alpha_{\max}) \). Utilising the condition of breakage \( (dS/d\varepsilon)_{\varepsilon^*} = 0 \) of the fibre bundle on Equation (12b) and then rearranging it, we obtain:

\[ \frac{dS(\alpha^*)}{da^*} \left[ \frac{1 - G(\alpha^*)}{\bar{S}(\alpha^*) g(\alpha^*)} \right] = 1 \]  

(15a)

\[ P^* = \bar{S}(\alpha)[1 - G(\alpha^*)] \]  

(15b)

The roots of Equations (15a) and (15b) are the respective values of \( \alpha^* \) and \( P^* \) under the assumption of symmetry in breaking force of fibres.

**Some relative variables and their uses**

We define the relative fibre breaking force \( y \) as a ratio of the fibre breaking force \( P \) to the average fibre breaking force \( \bar{P} \). Symbolically, \( y = P/\bar{P} \). So, \( dy = dP/\bar{P} \). Analogically, the relative fibre breaking strain \( z \) is defined as the ratio between the fibre breaking strain \( \alpha \) and the average fibre breaking strain \( \bar{\alpha} \). Symbolically, \( z = \alpha/\bar{\alpha} \). So, \( dz = d\alpha/\bar{\alpha} \). We also define the breaking force utilisation coefficient \( \eta_y \) as a ratio between the breaking force of the fibre bundle related to one fibre \( P^* \) and the average fibre breaking force \( \bar{P} \). Symbolically, \( \eta_y = P^*/\bar{P} \). Analogically, the breaking strain utilisation coefficient \( \eta_z \) is defined as a ratio between the breaking strain of the fibre bundle related to one fibre \( \varepsilon^* \) and the average fibre breaking strain \( \varepsilon^* \). Symbolically, \( \eta_z = \varepsilon^*/\varepsilon^* \).

The distribution of the relative fibre breaking points \( (y, z) \) is given by the probability density function \( w(y, z) \). From the theory of probability, we obtain

\[ w(y, z) dy dz = u(P, a) dP da. \]  

(16)

Then, following the above symbolism, Equation (16) takes the following form:

\[ w(y, z) = P \eta_y u(P, a). \]  

(17)

Using Equations (17) and (3) and also the definition of \( y \), the marginal probability density function of the relative fibre breaking strain \( h(z) \) can be expressed as

\[ h(z) = \int_{\eta_z}^{1} w(y, z) dy. \]  

(18)
\[ h(z) = \bar{\sigma} g(a). \] (18)

From the definitions of \( z \) and \( \eta_a \) obviously when \( a = a^* \) then \( z = \eta_a \). Hence Equation (18) can be written as

\[ h(\eta_a) = \sigma g(a^*). \] (19)

Substituting Equations (17) and (18) in the definition of the conditional probability density function of the relative fibre breaking force at a given relative fibre breaking strain \( \phi(y|z) \), and then comparing the resultant expression with (6), we obtain

\[ \phi(y|z) = \hat{P}(\eta_a) / P. \] (20)

Substituting Equation (20) and using the definition of \( y \) in the definition of the conditional average relative fibre breaking force at a given relative fibre breaking strain \( y(z) \), we obtain

\[ \frac{\sigma}{\eta_a} = \hat{P}(\eta_a) / P, \] (21)

It is already known that when \( a = a^* \) then \( z = \eta_a \). Hence Equation (21) takes the following form:

\[ \frac{\sigma}{\eta_a} = \hat{P}(\eta_a) / P. \] (22)

Now we define the relative fibre strain \( t \) as a ratio between the fibre strain \( e \) and the average fibre breaking strain \( \bar{e} \). Symbolically, \( t = e / \bar{e} \). So, \( dz = dt / \bar{e} \). Evidently, this is the relative strain of the fibre bundle also. Under this symbol, we can consider the average function \( \bar{S}(a) \) as shown below:

\[ \bar{S}(a) = \hat{P}(\eta_a) / P. \] (23)

where \( \zeta(t) = 1 / P \bar{S}(a) \).

Substituting \( \eta_a \) as the relative average force, the expression \( \bar{S}(a) \) can be written as

\[ S(a) = \hat{P}(\eta_a) / P. \] (24)

From the definitions of \( t \) and \( \eta_a \), it is obvious that when \( e = a \) then \( t = \eta_a \), and so Equation (23) can be expressed in another form, as follows:

\[ \hat{P}(\eta_a) = \hat{P}(\eta_a) / P. \] (25)

Now the following derivation is evident from Equation (23)

\[ \frac{d\bar{S}(a)}{da} = \frac{d\hat{P}(\eta_a)}{P} \zeta(t) / dt. \] (26)

It is already known that when \( e = a^* \) then \( t = \eta_a \), and so it is valid to write Equation (26) as

\[ \frac{d\bar{S}(a^*) / da^* = \hat{P}(\eta_a) / d\eta_a / d\eta_a. \] (27)

Now we define the relative average force per fibre in the bundle \( \sigma = S^*/P \). This takes the following forms under the three cases mentioned below:

Case 1 (no fibre is broken): From the definitions of \( t \) and \( z \), it is obvious that when \( e < a_{\text{min}} \) then \( t < \eta_{\text{min}} \). At first, substituting \( S \) from Equation (12a) into the definition of \( \sigma \), then utilising (23), (21), (24), (18), and the definition of \( z \), we obtain

\[ \sigma = \zeta(t) \int \frac{y(y)}{\zeta(y)} h(z) dz. \] (28a)

Case 2 (fibres with \( a < e \) are broken): From the definitions of \( t \) and \( z \), it is obvious that when \( e \in [a_{\text{min}}, a_{\text{max}}] \) then \( t \in [\eta_{\text{min}}, \eta_{\text{max}}] \). In analogy to the derivation of Equation (28a), we obtain

\[ \sigma = \zeta(t) \int \frac{y(y)}{\zeta(y)} h(z) dz. \] (28b)

Case 3 (all fibres are broken): Obviously, from the definitions of \( t \) and \( z \), when \( e > a_{\text{max}} \) then \( t > \eta_{\text{max}} \). Under this case, \( \eta^* = 0 \), hence obviously

\[ \sigma = 0 \] (28c)

Now utilising Equations (29a), (21), (24), (18), (22), and (19) into (13a) and then utilising the definitions of \( z \) and \( \eta_a \), we obtain

\[ \frac{d\zeta(y)}{d\eta_a} = \frac{y(y)}{\zeta(y)} h(z) / h(\eta_a). \] (29)

when \( \eta_a \geq \eta_{\text{min}} \).

At first, substituting \( P^* \) from Equation (13b) in the definition of \( \eta_{P} \) and then utilising (25), (21), (24), (18), and the definition of \( z \), we obtain

\[ \eta_{P} = \zeta(t) \int \frac{y(y)}{\zeta(y)} h(z) dz. \] (29b)

The roots of Equations (29a) and (29b) are the values of \( \eta_a \) and \( \eta_{P} \) respectively.

Under the assumption of symmetry in breaking forces of fibres, using Equations (21) and (24) the following expression is obtained:

\[ \gamma(\gamma) = \zeta(\gamma). \] (30a)

Substituting the random variable \( z \) by another random variable \( \eta_a \) in Equation (30a), we obtain

\[ \gamma(\eta_a) = \zeta(\eta_a). \] (30b)

Substituting Equation (30a) into (28a)-(28c) respectively, we obtain

\[ \sigma = \zeta(t), \text{ when } t < \eta_{\text{min}} \]. (31a)

\[ \sigma = \zeta(t)(1 - H(t)), \text{ when } t \in [\eta_{\text{min}}, \eta_{\text{max}}]. \] (31b)

\[ \sigma = 0, \text{ when } t > \eta_{\text{max}} \]. (31c)

where

\[ H(z) = \int h(z') dz'. \]

is the distribution function of \( z \). Substituting Equations (30a) and (30b) into (29a), we obtain the following expression:

\[ \frac{d\zeta(y)}{d\eta_a} = \frac{1 - H(\eta_a)}{k(\eta_a)} \] (32a)

Substituting Equation (30a) into (29b), we obtain

\[ \eta_{P} = \zeta(\eta_a)(1 - H(\eta_a)). \] (32b)

Equations (32a) and (32b) allow us to evaluate \( \eta_a \) and \( \eta_{P} \) respectively under the assumption of symmetry in breaking forces of fibres.

**Note:** Two ratios are shown at the left-hand side of Equation (32a): the first one represents force-strain relation, and the second one concerns the influence of the distribution of the relative fibre breaking points.

**Examples**

1) Assume the force-strain relation of fibres is linear. Then the average function must be linear also: \( \bar{S}(a) = (P, a^*) \). Comparing this expression with Equation (23) and utilising the definition of \( t \), we obtain

\[ \zeta(t) = t. \] So \( d\zeta(t) / dt = 1. \] Substituting the random variable \( t \) by another variable \( \eta_a \) into the relation \( \zeta(t) = t \), we obtain

\[ \zeta(\eta_a) = \eta_a. \] So \( d\zeta(\eta_a) / d\eta_a = 1. \)

2) Assume the fibre breaking points \((P, a)\) follow a two-dimensional Gaussian (normal) distribution \( a(P, a) \). From the theory of probability, we obtain that the marginal probability density function \( g(a) \) of fibre breaking strain must also be Gaussian with average \( \bar{a} \) and standard deviation \( s_a \); and the random variable \( z \) also follows Gaussian distribution, but with average 1 and standard deviation \( s_v \), where \( v_a = s_a / \bar{a} \). Evidently, \( v_a \) has the meaning of the coefficient of variation (CV) of the fibre breaking strain. So, the following expressions are valid:
Blended fibre bundle
Consider a blended fibre bundle consisting of \( M \) different components. The partial components are denoted by the serial number \( i = 1, 2, ..., m \) as a subscript. Assume each partial component has \( n_i \) fibres, and then the total number of fibres in the whole bundle is \( n = \sum n_i \).

The group of \( n_i \) fibres of one component can be understood as the \( i \)th partial bundle, consisting of fibres of only one component. If we symbolise the average force per fibre of the \( i \)th partial bundle by \( S_\alpha^i \) then the total force on all fibres of the \( i \)th partial bundle \( S_{\xi,i} \) is given by \( S_{\xi,i} = n_i S_\alpha^i \). Therefore, the resultant force on the whole bundle \( S_\xi \) is then \( n = \sum n_i \).

Hence the average force per fibre in the whole bundle \( S^* \) is obtained as
\[
S^* = S_\xi / n = \left( \sum n_i S_\alpha^i \right) / n .
\]

Obviously, the maximum value of force \( S^*_\xi \) is the breaking force of the whole bundle \( P_\xi \), and the strain \( \epsilon \) at which the relation \( S_{\xi,\epsilon} = P_\xi \) holds is the breaking strain of the whole bundle \( \alpha^* \).

Example
Consider a blended fibre bundle consisting of two components (\( M = 2 \)), where the fibres of each component satisfy the following assumptions:

1) Fibre force-strain relations are linear. Then the average force must be linear also: \( S_\xi(\epsilon) = P_\xi(\alpha) \epsilon \).

2) The fibre breaking strain follows a Gaussian distribution. Then the probability density function of fibre breaking strain is
\[
g_\alpha(\alpha) = \left( \frac{\sqrt{2\pi}}{s_\alpha} \right) \exp\left[ -\frac{(\alpha - \bar{\alpha})^2}{2s_\alpha^2} \right]
\]
and the corresponding distribution function is given by
\[
G_\alpha(\alpha) = \int g_\alpha(\alpha) \, d\alpha .
\]

3) The fibre breaking forces are symmetrical. Symbolically, \( P_\alpha(\alpha) = S_\alpha(\alpha) \).

Under these assumptions, the average force per fibre of the \( i \)th partial bundle \( S_\alpha^i \) can be obtained from Equation (15b), as follows:
\[
S_\alpha^i = \frac{1}{\Sigma} \left[ \frac{1}{1 - F(\alpha)} \right] = \frac{1}{\Sigma} .
\]

Using Equation (33) and the relation \( \alpha_i = (n_i/n)(P_i/\sigma_i) \), where \( \alpha_i \) is a characteristic parameter of the respective component, we obtain
\[
S^* = (n_1/n)S_1^* + (n_2/n)S_2^* = \alpha_1\epsilon_1\left[ 1 - G_1(\epsilon_1) \right] + \alpha_2\epsilon_2\left[ 1 - G_2(\epsilon_2) \right] .
\]

From reference [8], it is known that \( (n_1/n) = q_1/t_1 \), where \( q_1 \) is the mass portion of \( i \)th component such that \( \sum q_i = 1 \), \( t_i \) is the fineness of the \( i \)th component and \( t \) is the average fibre fineness. We consider another characteristic parameter \( \beta_i \) of the respective component as \( \beta_i = (q_i/t_i)/(P_i/\sigma_i) \). Then we obtain
\[
\frac{t}{\sqrt{\pi}} = n^* = T[\beta_1\epsilon_1\left[ 1 - G_1(\epsilon_1) \right] + \beta_2\epsilon_2\left[ 1 - G_2(\epsilon_2) \right] ] .
\]

The numerical solution of Equation (36) can give one to three roots. The ‘correct’ root, which corresponds to the actual breaking strain of the whole bundle \( \alpha^* \), is determined from the equation for calculation of breaking force. The breaking strain of the whole bundle \( \alpha^* \) and the breaking force of the whole bundle \( P_\xi \) are the coordinate of one point that lies on the curve of the fibre breaking strain distribution function of the whole bundle \( S_{\xi,\epsilon} = P_\xi \).
on the force-strain curve expressed by Equation (35). Therefore, we can write

$$P_{\Sigma} = T[\beta_1 G_1(\epsilon) + \beta_2 G_2(\epsilon)]$$

(37)

If Equation (36) has more roots, then the root leading to the highest value of $P_{\Sigma}$ found from Equation (37) is the required breaking strain of the whole bundle $\epsilon^*$. Evidently, from Equation (37), it is possible to obtain the breaking tenacity of the whole bundle $T_{\Sigma} = P_{\Sigma}/T$.

The above theory is illustrated with the help of two imaginary blended fibre bundles (FB 1 and FB 2), where each bundle consists of two different components. The fibres of each component have the following characteristics, as shown in Table 1. (In FB 1, component 1 is like polyester and component 2 is like cotton.) Figure 7a represents the tenacity-strain curves of FB 1 obtained from Equation (35) using the expressions for $\beta_a$, $g_a(\epsilon)$, $G_i(\epsilon)$ as considered before and the relation $g_1 + g_2 = 1$. The curves are almost bimodal, except for bundles with only one component, i.e., $g_1 = 0$ or $g_1 = 1$. Figure 7b illustrates the tenacity-strain curves of FB 1 on the basis of Hamburger’s theory ($s_{a1} \rightarrow 0$ and $s_{a2} \rightarrow 0$) [3].

The effect of variability in the breaking strain of fibres within a component on the force-strain behaviour of FB 1 can be understood by comparing these both sets of curves. By solving Equation (36) using the expressions for $\beta_a$, $g_a(\epsilon)$, $G_i(\epsilon)$ as considered before and the relation $g_1 + g_2 = 1$, we obtain the thick lines in Figures 8a and 8b showing the effect of blend ratio on the breaking tenacity and breaking strain of FB 1, respectively. The thin lines in Figures 8a and 8b are obtained on the basis of Hamburger’s theory ($s_{a1} \rightarrow 0$ and $s_{a2} \rightarrow 0$) [3]. Evidently, the shifting of the thick and thin lines is significant. (It is not true that all the fibres of one component break at the same time.) In the case of FB 2, where the fibres of one component differ from the other component only in terms of variability in fibre breaking strain, the effect of blend ratio on the breaking tenacity and breaking strain is shown in Figure 9. Evidently, the change of shape and the shifting of the thick and thin lines are significant. (The overlapping of the distributions of fibre breaking strain of the components is significant.)

### Conclusion

This work shows that it is possible to model the tensile behaviour of fibre bundles, where the constituent fibres possess different tensile behaviours. Extrapolating this fact into the model proves to be significant when predicting the tensile behaviour of the bundle; this behaviour is found to be different than that obtained from Hamburger’s theory. It is shown that the average force per fibre in the bundle, the breaking force utilisation coefficient, and the breaking strain utilisation coefficient depend only on the coefficient of variation of fibre breaking strain. It will be very useful to produce a set of blended fibre bundles and yarns under comparable parameters (material, technology, etc.), and experimentally verify the above theoretical model. Working out supplementary empirical corrections to this model will lead to a practical way for predicting the tensile behaviour of blended fibre bundles and yarns.

### Table 1. Characteristics of fibres in bundles FB 1 and FB 2.

<table>
<thead>
<tr>
<th>Fibre parameters</th>
<th>FB 1</th>
<th>FB 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Component 1</td>
<td>Component 2</td>
</tr>
<tr>
<td>Average breaking tenacity $\beta$, N/tex</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Average breaking strain $\epsilon$, %</td>
<td>30</td>
<td>8</td>
</tr>
<tr>
<td>Standard deviation of breaking strain $s$, %</td>
<td>0.015</td>
<td>0.024</td>
</tr>
<tr>
<td>CV of breaking strain $s$, %</td>
<td>5</td>
<td>30</td>
</tr>
</tbody>
</table>

### Figure 7. Tensile curves (tenacity-strain) of the fibre bundle FB 1: a) the presented theory b) Hamburger’s theory.

### Figure 8. Comparison between the presented theory and Hamburger’s theory with a view to the tensile behaviours of fibre bundle FB 1: a) Breaking tenacity vs. mass portion b) Breaking strain vs. mass portion.

### Figure 9. Comparison between the presented theory and Hamburger’s theory with a view to the tensile behaviours of fibre bundle FB 2: a) Breaking tenacity vs. mass portion b) Breaking strain vs. mass portion.
List of symbols

- $P$: Fibre breaking force
- $a$: Fibre breaking strain
- $u(P,a)$: Joint probability density function of fibre breaking force and fibre breaking strain
- $P_{\min}$: Minimum fibre breaking force
- $P_{\max}$: Maximum fibre breaking force
- $a_{\min}$: Minimum fibre breaking strain
- $a_{\max}$: Maximum fibre breaking strain
- $\bar{P}$: Average fibre breaking force
- $\bar{a}$: Average fibre breaking strain
- $g(a)$: Marginal probability density function of fibre breaking strain
- $G(a)$: Distribution function of fibre breaking strain
- $\psi(P,a)$: Conditional probability density function of fibre breaking force at a given fibre breaking strain
- $F(a)$: Conditional average fibre breaking force at a given fibre breaking strain
- $S$: Force on a fibre
- $s$: Strain on a fibre
- $S(e)$: Force on a fibre at a given fibre strain
- $S_i(e)$: Average force on fibres at a given fibre strain
- $k$: Fibre parameter
- $S(a)$: Force on a fibre at a given fibre breaking strain
- $S_i(a)$: Average force on fibres at a given fibre breaking strain
- $S_i(\epsilon)$: Average force on fibres at a given average fibre breaking strain
- $S^*$: Average force per fibre in a fibre bundle
- $P^*$: Breaking force of a fibre bundle related to one fibre
- $a^*$: Breaking strain of a fibre bundle related to one fibre
- $S_i(a^*)$: Average force on fibres at a given breaking strain of a fibre bundle related to one fibre
- $G(e)$: Distribution function of strain on fibres
- $G(a^*)$: Distribution function of breaking strain of a fibre bundle related to one fibre
- $g(a^*)$: Marginal probability density function of breaking strain of a fibre bundle related to one fibre
- $y$: Relative fibre breaking force
- $z$: Relative fibre breaking strain
- $\eta_p$: Fibre breaking force utilisation coefficient
- $\eta_a$: Fibre breaking strain utilisation coefficient
- $w(y,z)$: Probability density function of relative fibre breaking force and relative fibre breaking strain
- $h(z)$: Marginal probability density function of relative fibre breaking strain
- $\eta_\theta$: Marginal probability density function of fibre breaking strain utilisation coefficient
- $\eta(y,z)$: Conditional probability density function of relative fibre breaking force at a given relative fibre breaking strain
- $\gamma(y,z)$: Conditional average relative fibre breaking force at a given relative fibre breaking strain
- $\gamma(\eta)$: Conditional average fibre breaking force at a given fibre breaking strain utilisation coefficient
- $\bar{F}(a^*)$: Conditional average breaking force of a fibre bundle related to one fibre
- $\xi(t)$: Relative fibre strain
- $\zeta(t)$: Relative average function of fibre breaking strain
- $\zeta(a^*)$: Relative average function of fibre breaking strain
- $\sigma$: Relative average function per fibre in a fibre bundle
- $\bar{\sigma}$: Relative average function of partial bundle
- $z_{\min}$: Minimum relative fibre breaking strain
- $z_{\max}$: Maximum relative fibre breaking strain
- $H(t)$: Distribution function of relative fibre breaking strain
- $H(z)$: Distribution function of relative fibre breaking strain
- $H(\eta_i)$: Distribution function of fibre breaking strain utilisation coefficient
- $s_a$: Standard deviation of fibre breaking strain
- $v_a$: Coefficient of variation of fibre breaking strain
- $\mu_i, \mu_H$: Standardized random variables
- $f(u)$: Gaussian probability density function of $u$
- $F(a)$: Distribution function of $u$
- $\rho$: Standard deviation of fibre breaking force
- $\psi$: Correlation coefficient between fibre breaking force and fibre breaking strain
- $M$: No. of components (partial bundles) present in a blended fibre bundle
- $i$: Serial number denoting partial bundle, $i = 1, 2, ..., m$
- $n_i$: No. of fibres present in $i$th partial bundle
- $n$: Total no. of fibres present in a fibre bundle
- $S_i$: Average force per fibre of $i$th partial bundle
- $P_{\bar{z}}$: Breaking force of a fibre bundle
- $S_{\bar{z}}(c)$: Average force on fibres of $i$th partial bundle at a given fibre strain
- $P_{\bar{z}}$: Average breaking force of fibres of $i$th partial bundle
- $\bar{a}_i$: Average breaking strain of fibres of $i$th partial bundle
- $g(a)$: Marginal probability density function of breaking strain of fibres of $i$th partial bundle
- $s_{a,i}$: Standard deviation of breaking strain of fibres of $i$th partial bundle
- $G(a)$: Distribution function of breaking strain of fibres of $i$th partial bundle
- $\bar{P}(a)$: Conditional average breaking force of fibres of $i$th partial bundle at a given fibre breaking strain
- $S_{\bar{z}}(a)$: Average force on fibres of $i$th partial bundle at a given average fibre breaking strain
- $G(c)$: Distribution function of strain on fibres of $i$th partial bundle
- $\alpha_i, \beta_i$: Parameters characteristic to $i$th component
- $q_i$: Mass portion of $i$th component
- $f_i$: Fineness of fibres of $i$th component
- $T$: Average fiber fineness
- $T$: Fineness of the fiber bundle
- $g(a^*)$: Marginal probability density function of breaking strain of fibres of partial bundle

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References